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I declare this is my own work.

# A-level FURTHER MATHEMATICS

## Paper 1

Time allowed: 2 hours

### Materials

- You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)

### Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

### Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

### Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
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11	
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13	
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15	
<b>TOTAL</b>	



Answer **all** questions in the spaces provided.

**1** Find

$$\sum_{r=1}^{20} (r^2 - 2r)$$

Circle your answer.

[1 mark]

2450

2660

5320

43680

**2**

Given that  $z = 1 - 3i$  is one root of the equation  $z^2 + pz + r = 0$ , where  $p$  and  $r$  are real, find the value of  $r$ .

Circle your answer.

[1 mark]

-8

-2

6

10



3 The curve  $C$  has polar equation

$$r^2 \sin 2\theta = 4$$

Find a Cartesian equation for  $C$ .

Circle your answer.

[1 mark]

$$y = 2x$$

$$y = \frac{x}{2}$$

$$y = \frac{2}{x}$$

$$y = 4x$$

Turn over for the next question

Turn over ►



4 Show that the solutions to the equation

$$3 \tanh^2 x - 2 \operatorname{sech} x = 2$$

can be expressed in the form

$$x = \pm \ln(a + \sqrt{b})$$

where  $a$  and  $b$  are integers to be found.

You may use without proof the result  $\cosh^{-1} y = \ln(y + \sqrt{y^2 - 1})$

**[5 marks]**

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5 The matrix  $\mathbf{M}$  is defined by  $\mathbf{M} = \begin{bmatrix} 3 & 2 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Prove by induction that  $\mathbf{M}^n = \begin{bmatrix} 3^n & 3^n - 1 & -3^n + 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$  for all integers  $n \geq 1$

[5 marks]

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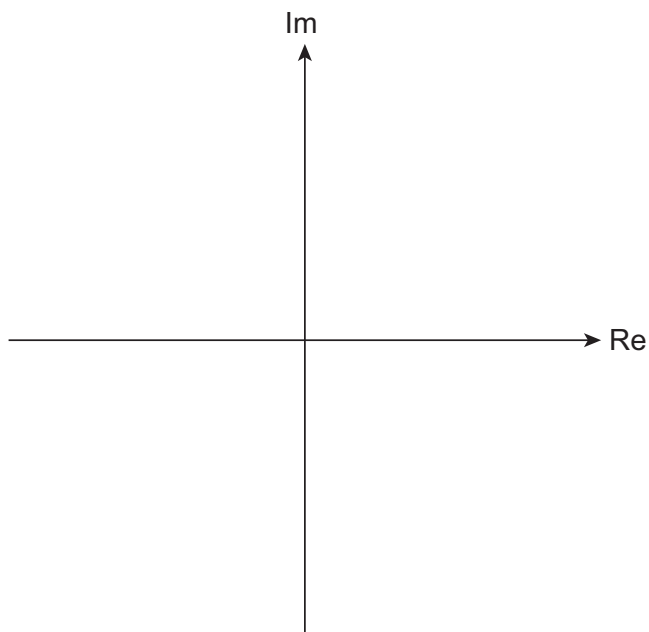






- 6 (b) (i)** Plot the four solutions to the equation in part (a) on the Argand diagram below and join them together to form a quadrilateral with one line of symmetry.

[2 marks]



- 6 (b) (ii)** Show that the area of this quadrilateral is  $\frac{\sqrt{15}}{2}$  square units.

[1 mark]

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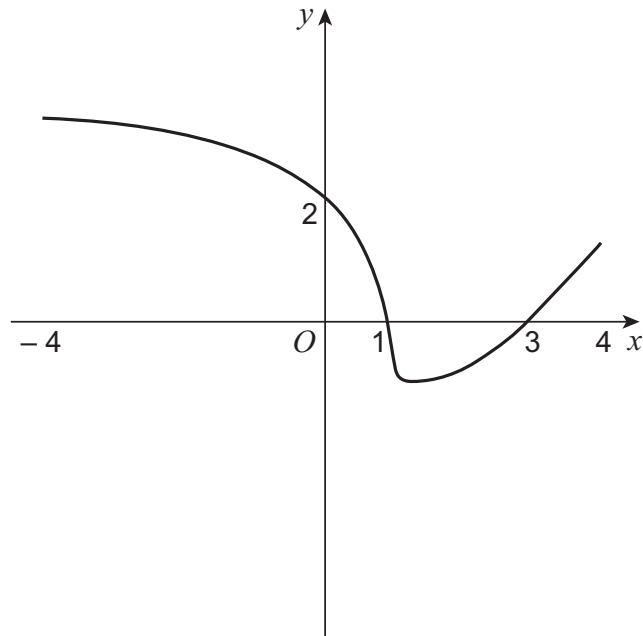
Turn over ►



7 The diagram below shows the graph of  $y = f(x)$  ( $-4 \leq x \leq 4$ )

The graph meets the  $x$ -axis at  $x = 1$  and  $x = 3$

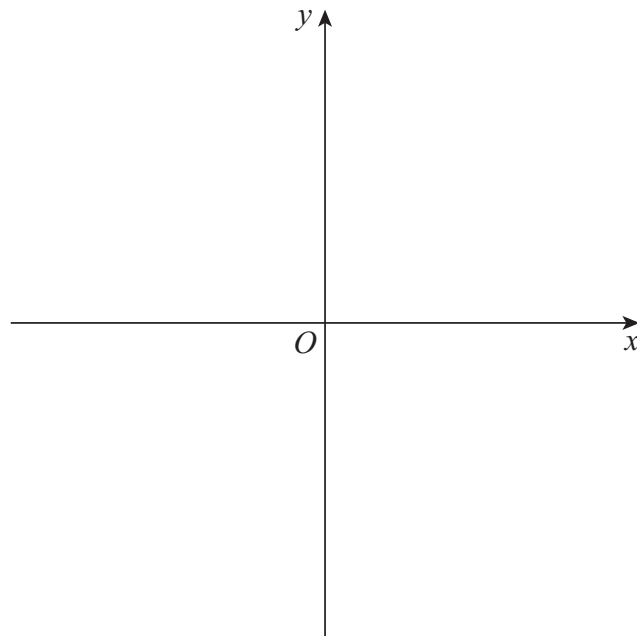
The graph meets the  $y$ -axis at  $y = 2$



7 (a) Sketch the graph of  $y = |f(x)|$  on the axes below.

Show any axis intercepts.

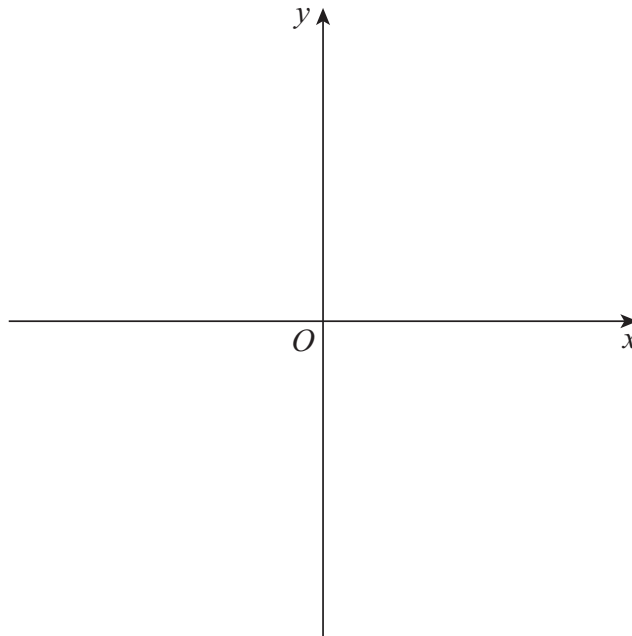
[2 marks]



**7 (b)** Sketch the graph of  $y = \frac{1}{f(x)}$  on the axes below.

Show any axis intercepts and asymptotes.

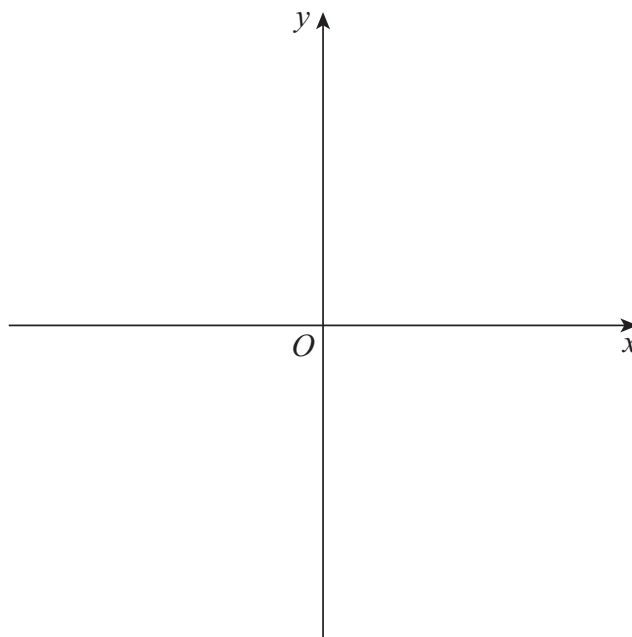
**[3 marks]**



**7 (c)** Sketch the graph of  $y = f(|x|)$  on the axes below.

Show any axis intercepts.

**[2 marks]**



**Turn over ►**



**8** A particle of mass 4 kg moves horizontally in a straight line.

At time  $t$  seconds the velocity of the particle is  $v \text{ m s}^{-1}$

The following horizontal forces act on the particle:

- a constant driving force of magnitude 1.8 newtons
- another driving force of magnitude  $30\sqrt{t}$  newtons
- a resistive force of magnitude  $0.08v^2$  newtons

When  $t = 70$ ,  $v = 54$

Use Euler's method with a step length of 0.5 to estimate the velocity of the particle after 71 seconds.

Give your answer to **four** significant figures.

**[6 marks]**

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11 (b) (ii) Hence find the shortest distance of the plane  $\Pi_1$  from the origin.

[1 mark]

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11 (c) The points  $A(4, -1, -1)$ ,  $B(1, 5, -7)$  and  $C(3, 4, -8)$  lie in the plane  $\Pi_2$

Find the angle between the planes  $\Pi_1$  and  $\Pi_2$ , giving your answer to the nearest  $0.1^\circ$

[4 marks]

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12 The matrix  $\mathbf{A} = \begin{bmatrix} 1 & 5 & 3 \\ 4 & -2 & p \\ 8 & 5 & -11 \end{bmatrix}$ , where  $p$  is a constant.

12 (a) Given that  $\mathbf{A}$  is a non-singular matrix, find  $\mathbf{A}^{-1}$  in terms of  $p$ .

State any restrictions on the value of  $p$ .

**[6 marks]**

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**12 (b)** The equations below represent three planes.

$$x + 5y + 3z = 5$$

$$4x - 2y + pz = 24$$

$$8x + 5y - 11z = -30$$

**12 (b) (i)** Find, in terms of  $p$ , the coordinates of the point of intersection of the three planes.

**[4 marks]**

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13 The transformation S is represented by the matrix  $\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$

The transformation T is a translation by the vector  $\begin{bmatrix} 0 \\ -5 \end{bmatrix}$

Kamla transforms the graphs of various functions by applying first S, then T.

Leo says that, for some graphs, Kamla would get a different result if she applied first T, then S.

Kamla disagrees.

State who is correct.

Fully justify your answer.

**[3 marks]**

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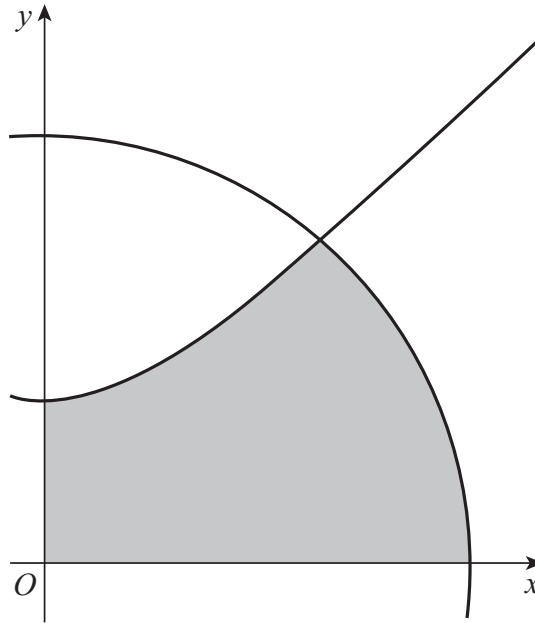


14

The hyperbola  $H$  has equation  $y^2 - x^2 = 16$

The circle  $C$  has equation  $x^2 + y^2 = 32$

The diagram below shows part of the graph of  $H$  and part of the graph of  $C$ .



Show that the shaded region in the first quadrant enclosed by  $H$ ,  $C$ , the  $x$ -axis and the  $y$ -axis has area

$$\frac{16\pi}{3} + 8 \ln \left( \frac{\sqrt{2} + \sqrt{6}}{2} \right)$$

[12 marks]

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**15** In this question use  $g = 9.8 \text{ m s}^{-2}$

A particle  $P$  of mass  $m$  is attached to two light elastic strings,  $AP$  and  $BP$ .

The other ends of the strings,  $A$  and  $B$ , are attached to fixed points which are 4 metres apart on a rough horizontal surface at the bottom of a container.

The coefficient of friction between  $P$  and the surface is 0.68

- When the extension of string  $AP$  is  $e_A$  metres, the tension in  $AP$  is  $24me_A$
- When the extension of string  $BP$  is  $e_B$  metres, the tension in  $BP$  is  $10me_B$
- The natural length of string  $AP$  is 1 metre
- The natural length of string  $BP$  is 1.3 metres



**15 (a)** Show that when  $AP = 1.5$  metres, the tension in  $AP$  is equal to the tension in  $BP$ .

**[1 mark]**

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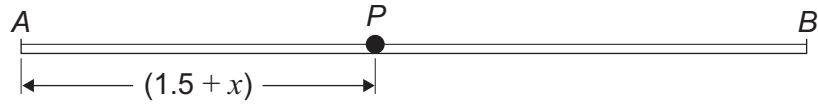




**15 (b)**

$P$  is held at the point between  $A$  and  $B$  where  $AP = 1.9$  metres, and then released from rest.

At time  $t$  seconds after  $P$  is released,  $AP = (1.5 + x)$  metres.



Show that when  $P$  is moving towards  $A$ ,

$$\frac{d^2x}{dt^2} + 34x = 6.664$$

**[3 marks]**

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**Question 15 continues on the next page**

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**15 (c)**

The container is then filled with oil, and  $P$  is again released from rest at the point between  $A$  and  $B$  where  $AP = 1.9$  metres.

At time  $t$  seconds after  $P$  is released, the oil causes a resistive force of magnitude  $10mv$  newtons to act on the particle, where  $v \text{ m s}^{-1}$  is the speed of the particle.

Find  $x$  in terms of  $t$  when  $P$  is moving towards  $A$ .

**[9 marks]**

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