AS
FURTHER MATHEMATICS
7366/2S

Paper 2 Statistics

Mark scheme

June 2021

Version: 1.0 Final Mark Scheme

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
sf	significant figure(s)
dp	decimal place(s)

Examiners should consistently apply the following general marking principles:

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Q	Marking instructions	AO	Marks	Typical solution
1	Circles correct answer	1.1b	B1	104
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
2	Circles correct answer	1.1b	B1	Po(18)
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
3(a)	Uses the formula for the mean of a discrete uniform distribution to form an equation Condone one sign error	1.1a	M1	$\frac{n+1}{2}=8$ $n+1=16$
	Completes a rigorous algebraic proof by solving the equation to show that $n = 15$	2.1	R1	$n=15$
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
3(b)	Obtains correct value of P($X > 4$)	1.1b	B1	$\text{P}(X>4)=11\times\frac{1}{15}=\frac{11}{15}$
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
3(c)	Uses the formula for the variance of a discrete uniform distribution PI	1.1a	M1	$\text{Var}(X)=\frac{15^2-1}{12}$ $=\frac{56}{3}$
	Obtains $\text{Var}(X)=\frac{56}{3}$	1.1b	A1	
	Total		**2**	

	Question total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
4(a)	Obtains correct z value AWRT 1.81 PI by a correct value of $\sqrt{n}$ or n	1.1b	B1	$z = 1.8119$ $0.02715 = 1.8119 \times \frac{0.3}{\sqrt{n}}$
	Uses formula for the full or half width of a confidence interval using their z-value to obtain an equation and attempts to solve, reaching at least $\sqrt{n} =$ PI	1.1a	M1	$\sqrt{n} = 1.8119 \times \frac{0.3}{0.02715}$ $n = 401$
	Obtains the correct value of n = 400 or 401 depending on accuracy given for z	1.1b	A1	
	Total		**3**	

Q	Marking instructions	AO	Marks	Typical solution
4(b)	Obtains correct confidence interval	1.1b	B1	(10.773, 10.827)
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
4(c)(i)	Evaluates the model by comparing the proposed population mean with their confidence interval found in part **(b)** and concludes that the null hypothesis will be rejected	3.5a	E1F	The null hypothesis will be rejected as 10.7 is outside the confidence interval
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
4(c)(ii)	Interprets Type II error in context	3.2a	E1	It is accepted that the mean distance travelled by the player in a football match is 10.7 kilometres when it is not
	Total		**1**	

	Question total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
5(a)	States correct modal score	1.1b	B1	Modal score = 10
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
5(b)	Models the situation with a discrete random variable (PI)	3.1b	M1	x | 0 | 5 | 10 | 20 $P(X \leq x)$ | 0.25 | 0.55 | 0.9 | 1 Median score = 5
	Obtains correct median score	3.2a	A1	
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
5(c)	Uses the formula for the mean of a discrete random variable to calculate the mean score or adjusts model to directly model prize money	3.4	M1	Mean score = $0 \times 0.25 + 5 \times 0.3 + 10 \times 0.35 + 20 \times 0.1$ $= 7$
	Obtains correct mean prize money	3.2a	A1	Mean prize money = 7×100 = £700
	Total		**2**	

	Question total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
6(a)	Integrates $\int_0^m \frac{1}{114}(4x+7)\,\mathrm{d}x$ and equates with 0.5	1.1a	M1	$\int_0^m \frac{1}{114}(4x+7)\,\mathrm{d}x = 0.5$ $\frac{1}{114}(2m^2+7m) = 0.5$ $\frac{1}{57}m^2 + \frac{7}{114}m - 0.5 = 0$ m = 3.8680512 m = 3.87 (3 sig fig)
	Obtains correct quadratic equation in terms of the median equal to zero PI	1.1a	M1	
	Completes rigorous argument to show that median is 3.87 to 3 significant figures. A more accurate answer must be seen and the other solution from the quadratic equation must be rejected if seen.	2.1	R1	
	Total		**3**	

Q	Marking instructions	AO	Marks	Typical solution
6(b)	Uses an integral of $\mathrm{f}(x)$ with one limit of 2 PI	1.1a	M1	$\mathrm{P}(X > 2) = \int_2^6 \frac{2}{57}x + \frac{7}{114}\,\mathrm{d}x$ $= \frac{46}{57}$
	Obtains the correct exact value of P(X > 2)	1.1b	A1	
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
6(c)(i)	Uses the general formula for $\mathrm{E}(\mathrm{f}(Y))$ to obtain $\mathrm{E}\left(\frac{1}{Y}\right)$	1.1a	M1	$\mathrm{E}\left(\frac{1}{Y}\right) = \int_1^3 y^{-1}\left(\frac{1}{2}y^2 - \frac{1}{6}y^3\right)\mathrm{d}y$ $= \frac{5}{9}$
	Uses the general formula for $\mathrm{E}(\mathrm{f}(Y))$ to obtain $\mathrm{E}\left(\frac{1}{Y^2}\right)$	1.1a	M1	$\mathrm{E}\left(\frac{1}{Y^2}\right) = \int_1^3 y^{-2}\left(\frac{1}{2}y^2 - \frac{1}{6}y^3\right)\mathrm{d}y$ $= \frac{1}{3}$
	Uses the formula for the variance to obtain an expression for $\mathrm{Var}\left(\frac{1}{Y}\right)$	1.1a	M1	$\mathrm{Var}\left(\frac{1}{Y}\right) = \mathrm{E}\left(\frac{1}{Y^2}\right) - \left(\mathrm{E}\left(\frac{1}{Y}\right)\right)^2$ $= \frac{1}{3} - \left(\frac{5}{9}\right)^2$
	Completes a rigorous argument to obtain the given value of $\mathrm{Var}\left(\frac{1}{Y}\right)$	2.1	A1	$= \frac{2}{81}$
	Total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
6(c)(ii)	Uses correct general formula for the sum of linear functions of independent variables and to obtain an expression for $\mathrm{Var}\left(AX - \frac{B}{Y}\right)$ Condone sign error	1.1a	M1	$\mathrm{Var}\left(2X - \frac{3}{Y}\right)$ $= 2^2\,\mathrm{Var}(X) + 3^2\,\mathrm{Var}\left(\frac{1}{Y}\right)$ $= 4 \times \frac{939}{361} + 9 \times \frac{2}{81}$ $= 10.6$
	Obtains the correct value of $\mathrm{Var}\left(2X - \frac{3}{Y}\right)$ AWRT 10.6	1.1b	A1	
	Total		**2**	

	Question total		**11**	

<table>
<tr><th>Q</th><th>Marking instructions</th><th>AO</th><th>Marks</th><th>Typical solution</th></tr>
<tr><td rowspan="10">7(a)</td><td>States both hypotheses using correct language. Variables need to be stated in at least the null hypothesis</td><td>2.5</td><td>B1</td><td rowspan="10">H_0: There is no association between employee and number of errors per day
H_1: There is an association between employee and number of errors per day

<table><tr><td>E</td><td>0</td><td>1</td><td>2</td><td>3+</td></tr><tr><td>A</td><td>13</td><td>7</td><td>17.5</td><td>2.5</td></tr><tr><td>B</td><td>13</td><td>7</td><td>17.5</td><td>2.5</td></tr></table>
<table><tr><td>2+</td><td>O</td><td>E</td></tr><tr><td>A</td><td>22</td><td>20</td></tr><tr><td>B</td><td>18</td><td>20</td></tr></table>
$\sum\frac{(O-E)^2}{E}$ =
$\frac{(8-13)^2}{13}+\frac{(18-13)^2}{13}$ +
$\frac{(10-7)^2}{7}+\frac{(4-7)^2}{7}$ +
+ $\frac{(22-20)^2}{20}+\frac{(18-20)^2}{20}$
= 6.8

χ^2 cv for 2 df = 5.991

5.991 < 6.8

Reject H_0

Some evidence to suggest/support that there is an association between employee and number of errors per day</td></tr>
<tr><td>Translate situation into expected contingency table for χ^2 model</td><td>3.3</td><td>M1</td></tr>
<tr><td>Combines columns for “2” and “3 or more” for both observed and expected values</td><td>1.1b</td><td>A1</td></tr>
<tr><td>Uses χ^2 model to calculate test statistic</td><td>3.4</td><td>M1</td></tr>
<tr><td>Obtains correct value of $\sum\frac{(O-E)^2}{E}$
AWRT 6.8</td><td>1.1b</td><td>A1</td></tr>
<tr><td>Obtains correct critical value for the test
AWRT 6.0
Or corresponding probability of test statistic
AWRT 0.03</td><td>1.1b</td><td>B1</td></tr>
<tr><td>Evaluates χ^2-test statistic by comparing the critical value with the test statistic or the probability with 0.05</td><td>3.5a</td><td>R1</td></tr>
<tr><td>Infers H_0 rejected
FT ‘their’ comparison using the χ^2 model.</td><td>2.2b</td><td>E1F</td></tr>
<tr><td>Concludes in context
(The conclusion must not be definite.)
FT their incorrect acceptance of H_0 if stated or ‘their’ comparison if not</td><td>3.2a</td><td>E1F</td></tr>
</table>

	Total		**9**	

Q	Marking instructions	AO	Marks	Typical solution
7(b)	Explains reasoning by considering ($O - E$) or $\frac{(O-E)^2}{E}$ to identify largest sources of association	2.4	E1	Largest sources of association employee A/0 errors and employee B/0 errors $\frac{(O-E)^2}{E}$ = 1.9…
	Interprets main source of association in context	3.2a	E1	Employee A makes no errors per day less often than expected
	Total		**2**	

	Question total		**11**	

	Paper total		**40**	