

2021 Assessment resources

GCSE Mathematics

Algebra – Higher

Answers and commentaries

The question numbers in this resource reflect the question numbers from the original papers and match the question numbers in the corresponding 2021 assessment materials.

Question 5

5 Solve $5(x + 3) < 60$

[2 marks]

Answer _____

Student A

Solve $5(x + 3) < 60$

$$\begin{array}{l} 5x + 15 < 60 \\ 5x < 45 \\ x = 9 \end{array}$$

Answer $x = 9$

Commentary

This student has done their working perfectly in the first two lines, either of which would get them the first mark. Unfortunately, when giving their answer they have changed the inequality sign to an equals sign, which loses the second mark. Note that the answer alone with no working would have received one mark, as there is an SC1 on this question for identifying 9 as the value with incorrect

signage. This is because so many students find it difficult to work algebraically with inequality signs but can actually process the values correctly.

1 mark

Student B

Solve $5(x + 3) < 60$

$$\begin{array}{r} 5(x + 3) = 60 \\ 5x + 15 = 60 \\ -15 \quad -15 \\ \hline 5x = 45 \\ \div 5 \quad \div 5 \\ \hline x = 9 \end{array}$$

Answer 9

Commentary

This student has failed to use the correct inequality sign throughout, and therefore doesn't qualify for the method or accuracy marks. However, they do receive the SC1 mark as they have identified 9 as the final value, which is correct. Many students feel more comfortable working with an equals sign, and then revert to the correct inequality sign for their answer. While this achieves full marks if done correctly it is dangerous, as forgetting to revert (as here) or reverting incorrectly can lose all but one of the marks, and all of them if there is an arithmetic error.

1 mark

Student C

Solve $5(x + 3) < 60$

$$\begin{array}{rcl} 5(x + 3) & = & 60 \\ 5x + 15 & = & 60 \\ 5x & = & 75 \\ x & = & 15 \end{array}$$

Answer 15

Commentary

This is an example of a student working with '=' who loses a mark because of it. The second line shows a correct expansion, which would have attracted the first mark with the correct sign, but loses it because the '=' sign has been used. They have then added 15 to 60 instead of subtracting, so the chance of SC1 is also lost.

0 marks

Question 10

No examples available

Question 5

5 Solve $4(3x - 2) = 2x - 5$

[3 marks]

$x =$ _____

Student A

Solve $4(3x - 2) = 2x - 5$

$$\begin{aligned}4(3x - 2) &= 2x - 5 \\12x - 8 &= 2x - 5 \\12x - 2x - 8 &= 5 \\10x - 8 &= 5 \\10x &= 13 \\x &= \frac{10x - 3}{10}\end{aligned}$$

Commentary

This student has correctly expanded the brackets for the first mark, but the equals sign has then changed into a subtraction sign, costing them the second mark. They don't seem to know how to complete their response.

1 mark

Student B

Solve $4(3x - 2) = 2x - 5$

[3 m]

$$\frac{4 \times (3 \times ? - 2)}{-2} = \frac{2 \times x - 5}{-2}$$

$$\cancel{3 \times 3} \quad 2 \times (3 \times ? - 2) = \cancel{2} - 5$$

$$3 \times 10 = 30 - 2 = 28 \times 4 = 112$$

$x =$ _____

Commentary

This student has embedded a question mark instead of x into the left hand side of the equation and then substituted 10 for it. Presumably they have realised that 112 is far too high for the right hand side and given up. Note that trial and improvement is all or nothing in terms of marks.

0 marks

Student C

Solve $4(3x - 2) = 2x - 5$

$$\begin{array}{r} 12x - 8 = 2x - 5 \\ +2x \quad -2x \end{array}$$

$$\begin{array}{r} 14x - 8 = -5 \\ -8 \quad +8 \end{array}$$

$$\begin{array}{r} 14x = 3 \\ 4 \end{array}$$

$$x = \frac{3}{14}$$

$$x = \frac{3}{14}$$

or
0.2142857

Turn over for the next question

Commentary

This student has expanded the brackets correctly for the first mark, but in collecting like terms has added $2x$ instead of subtracting, losing the second mark. In this scheme there is a follow through from one error in the working, so correctly going from $14x = 3$ to $x = \frac{3}{14}$ scores the final mark.

2 marks

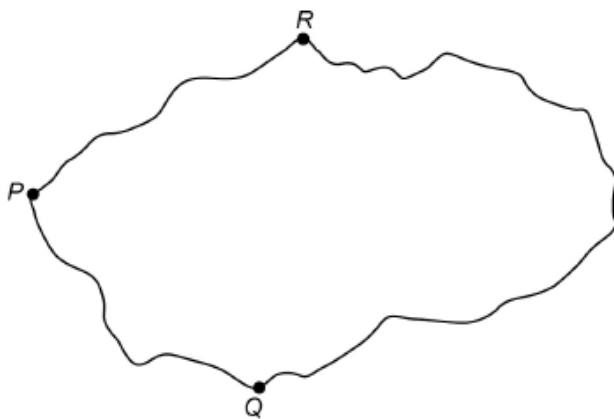
Question 9

- 9 Towns P , Q and R are connected by roads PQ , PR and QR .

PR is 10 km longer than PQ .

QR is twice as long as PR .

The total length of the three roads is 170 km



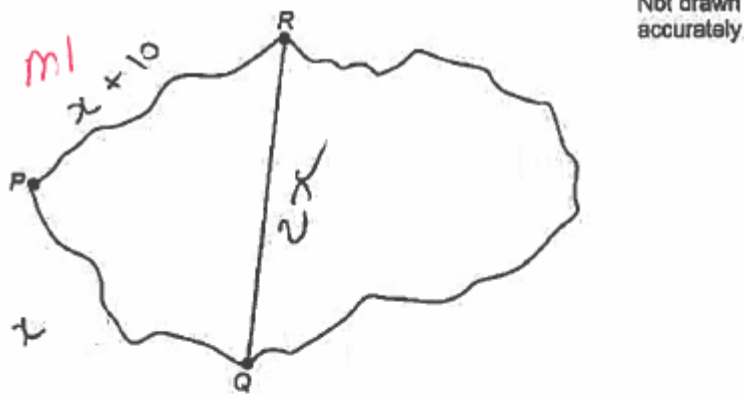
Not drawn
accurately

Work out the length of PQ .

[4 marks]

Answer _____ km

Student A



Work out the length of PQ.

m1 dep $x + x + 10 + 2x = 170$

$$4x + 10 = 170$$

$$\begin{array}{r} -10 \\ -10 \end{array}$$

m0 $4x = 160$

$$x = \frac{160}{4}$$

$$x = 40$$

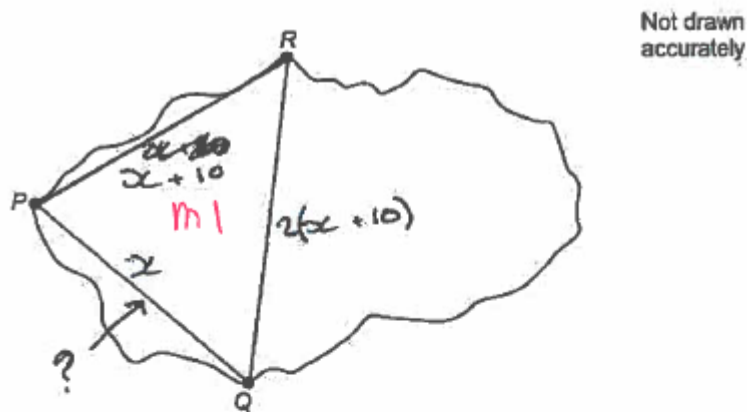
Answer AO 40 km

Commentary

The student has correctly given the expression ' $x + 10$ ' for PR, which gets them the first mark. Unfortunately, they have taken QR to be twice as long as PQ rather than PR. This still allows them to score the second mark, for a correct equation using their terms, but they can't go any further as the third mark stipulates that values must be correct.

2 marks

Student B



Work out the length of PQ.

$$2(x+10) = 2x + 20$$

$$2x + 20 + x + 10 + x = 4x + 30$$

$$170 + 30 = 200 \quad m0$$

$$200 \div 4 = 50$$

$$x = 50 \quad m0$$

~~170 + 30 = 200~~
~~200 \div 4 = 50~~

$$PQ = 50$$

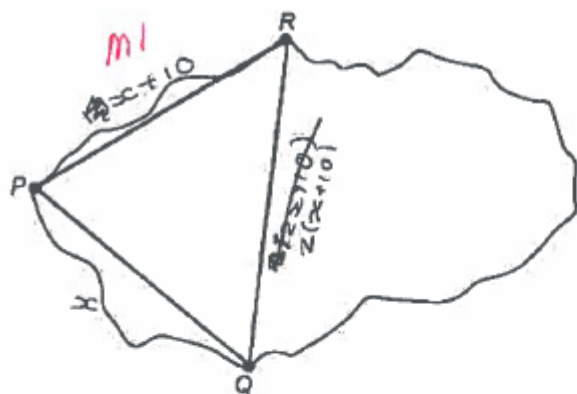
Answer 50 AO km

Commentary

This student has achieved the first mark for either $x + 10$ or $2(x + 10)$. They should have achieved the second mark, but have failed to set up an acceptable equation, adopting a semi-algebraic, semi-numerical approach. This would not have mattered if they had correctly arrived at the answer 35, but adding 30 to 170 instead of subtracting has cost them the last three marks.

1 mark

Student C



Not drawn accurately

Work out the length of PQ.

[4 m]

$$\begin{aligned}
 & x+10 + x + 2(x+10) = 170 \text{ km} \\
 & 2x+12 + 2x+20 = 170 \text{ km} \\
 & 4x+32 = 170 \text{ km} \\
 & 4x = 138 \text{ km} \\
 & x = 34.5 \text{ km} \\
 & \text{Answer } 34.5 \text{ km}
 \end{aligned}$$

Commentary

This student should have scored full marks, but has made an error in the second line which has cost them the last two marks, writing '12' instead of '10'. This may have been simply a transcription error, but as working must be correct for the last two marks that is irrelevant. This shows the benefit of checking your work, as it is likely the student would have spotted their error with a fairly rudimentary check.

2 marks

Question 26

26

Expand and simplify

$(x - 4)(2x + 3y)^2$

[4 marks]

Answer

Student A

Expand and simplify $(x-4)(2x+3y)^2$

[4 marks]

$$\begin{array}{l}
 \cancel{x \times 2x} = \cancel{2x^2} \\
 \cancel{-4 \times} \quad \cancel{4 \times 3y} \\
 \cancel{x \times 2x} = \cancel{2x^2} \\
 \cancel{x \times 3y^2} = \cancel{3xy^2} \\
 \text{A} \quad (x-4)(2x+3y)^2 \\
 \\
 \begin{array}{ll}
 x \times 2x^2 = \cancel{2x^2} 2x^4 & 2x^4 \quad x \times 2x^2 \\
 x \times 3y^2 = 3xy^2 & 8x^2 \\
 4 \times 2x^2 = 8x^2 & 10x^4 + 3x = 13x^4 \\
 4 \times 3y^2 = 12y^2 & 12y^4
 \end{array} \\
 \\
 \text{Answer } \quad 13x^4 + 12y^4
 \end{array}$$

Commentary

The student has expanded the squared bracket incorrectly to get only two terms, so no marks are possible.

0 marks

Student B

Expand and simplify $(x-4)(2x+3y)^2$

$$\begin{array}{r} \cancel{2x^2} \quad x^2 \quad 2x^2 \\ 2x^2 \quad -4^2 \quad 3y^2 \\ -8x \\ 3yx^2 \quad 2x^4 \\ -12y^2 \quad -8x^2 \\ 3yx^2 \\ 12y^2 \end{array}$$

Answer $2x - 8x + 3yx - 12y$

Commentary

There appears to be no worthwhile work here, but the expression on the answer line is actually worth M1 on alternative scheme 2, as it is an expansion of $(x-4)(2x-3)$ with four terms and only one error.

1 mark

Student C

Expand and simplify

$$(x - 4)(2x + 3y)^2$$

[4 marks]

$$\begin{array}{r} 2x \quad 3y \\ 2x \quad 2x^2 \quad 6xy \\ 3y \quad 6xy \quad 9y^2 \end{array}$$

$$(x - 4) \times (2x^2 + 12xy + 9y^2)$$

$$\begin{array}{r} 2x^2 + 12xy \quad 9y^2 \\ x \quad 2x^3 \quad 12xy^2 \quad 9xy^2 \\ -4 \quad -8x^2 \quad -48xy \quad -36y^2 \end{array}$$

Answer $2x^3 + 21xy^2 - 8x^2 - 48xy - 36y^2$

Commentary

The student has expanded the squared bracket first, and scores M1, but not A1 as there is one error ($2x^2$ should be $4x^2$). When following through to multiplying their expansion by $(x - 4)$, the multiplication by -4 is correct, again scoring M1, but when multiplying by x , $12xy^2$ should be $12x^2y$, so the last mark is not awarded.

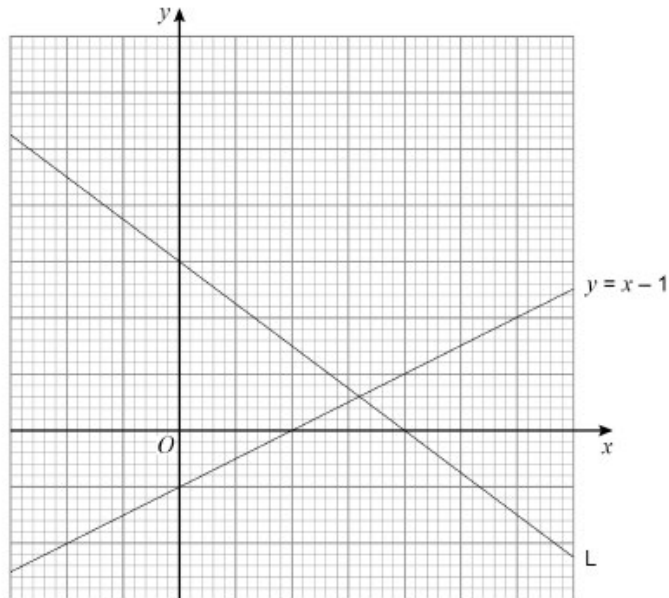
2 marks

Question 2

Please see the mark scheme

Question 15

- 15 Here is line L and the graph of $y = x - 1$
The scales of the axes are not shown.

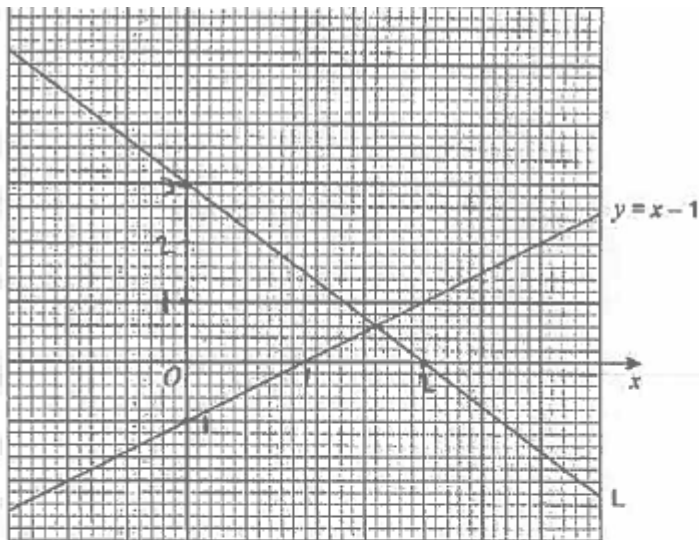


Work out the equation of line L.

[4 marks]

Answer

Student A



Work out the equation of line L.

[4 marks]

$$y = mx + c$$

$$\text{gradient} = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{3}{2}$$

$$m = 1.5$$

$$y = 1.5x + c$$

$$3 = 1.5x + c$$

$$3 = 1.5 \times 2 + c$$

$$c = 0$$

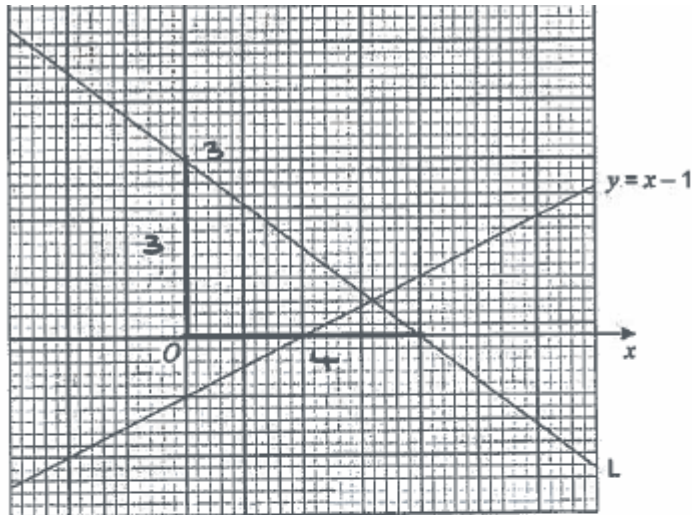
Answer $y = 1.5x$

Commentary

This mark scheme is from B4 rather than a method build-up, so we are looking for particular things here. The student has correctly worked out the scaling on both axes, as marked on the grid, so they get at least B2. They have also correctly worked out 3 and 2, but unfortunately have failed to realise that the graph is decreasing, and therefore 3 should really be -3 . This prevents them accessing the last two marks.

2 marks

Student B



Work out the equation of line L.

$$\frac{\text{rise}}{\text{run}} = \frac{3}{4} = 0.75$$

$$y = 0.75x + 3$$

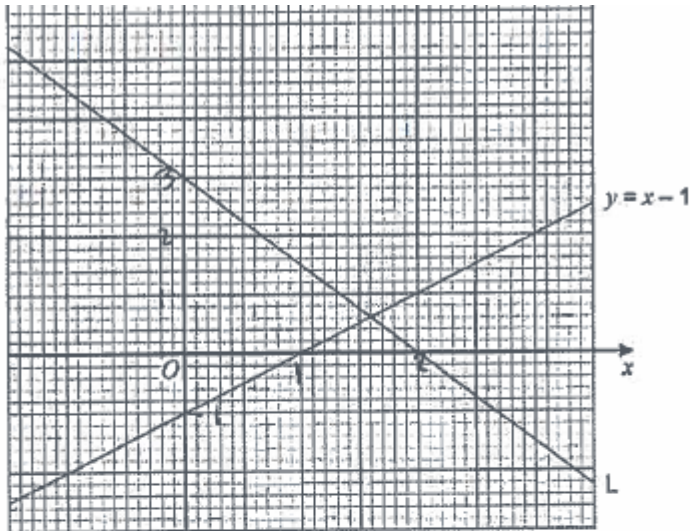
Answer $y = -0.75x + 3$

Commentary

This student has failed to realise that the scales are not unitary, which prevents them scoring on the B mark scheme. However, there is an SC2 mark available for students who assume the scales are unitary and go on to work out the correct equation for those scales. This student has done that, and even though the negative sign is missing from the initial working it is clearly given in the answer.

2 marks

Student C



Work out the equation of line L.

[4]

$$y = mx + c \quad \frac{dy}{dx}$$

Answer $y = 2x + 3$

Commentary

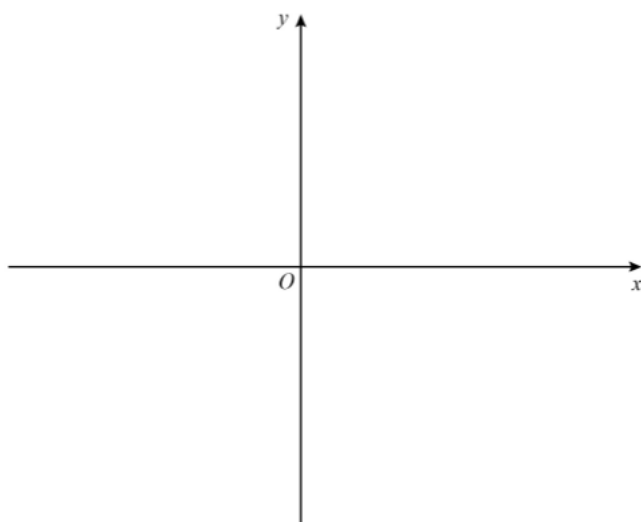
At first sight this response seems to have little or no merit, with the standard equation of a straight line written along with the irrelevant dy/dx and then an incorrect answer. However, we must check the scales in this question, and the student has shown that they understand the scaling, so receives B2.

2 marks

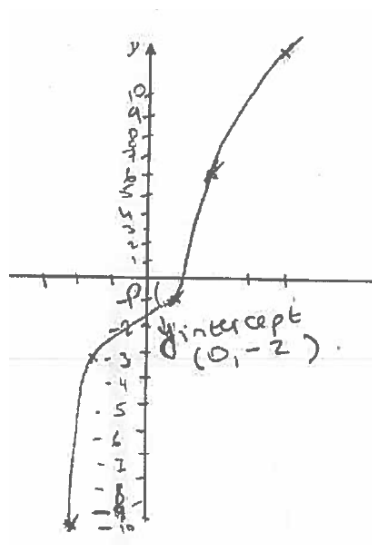
Question 8

- 8 On the axes, sketch the curve $y = x^3 - 2$
You **must** show the coordinates of the y-intercept.

[2 marks]



Student A

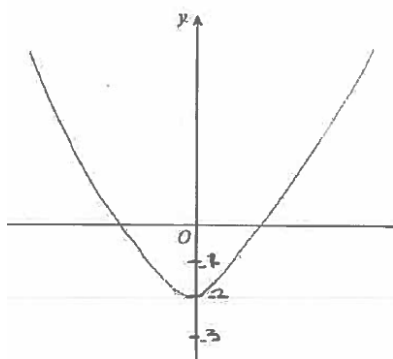


Commentary

The student is awarded B1, as the y-intercept is correctly labelled and the graph meets the criterion as it starts in the 1st quadrant and finishes in the 3rd quadrant, passes through the 4th quadrant and has no section with a negative gradient. Full marks cannot be awarded as the last part of the graph is convex rather than concave.

1 mark

Student B

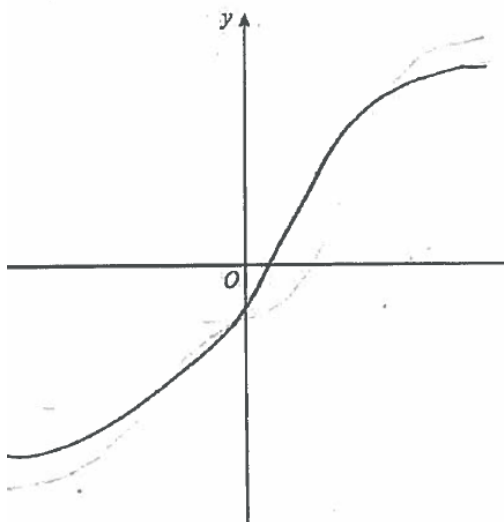


Commentary

The student has correctly identified the y -intercept, but that alone is not enough for a mark. Their curve has a negative gradient to the left of the turning point, and any curve with a negative gradient at any point scores zero.

0 marks

Student C



Commentary

At first sight it appears that this student might receive B1, as the curve passes through the correct quadrants and has no point with a negative gradient (benefit of the doubt at the ends). However, for B1 the y -intercept must be identified, and this has not happened here.

0 marks

Question 21

21 Here are the first four terms of a quadratic sequence.

11 26 45 68

Work out an expression for the n th term.

[3 marks]

Answer _____

Student A

21 Here are the first four terms of a quadratic sequence.

7 11 20 26 39 45 58 68

Work out an expression for the n th term.

-4 -4 +4 +4

$4n - 7 / 4n + 7$

Answer $4n - 7 / 4n + 7$

Commentary

The student has got into a bit of a tangle when dealing with the first and second differences, and loses the first mark as the '4's are not consistent with their positive and negative signs. The student makes no headway from there.

0 marks

Student B

Here are the first four terms of a quadratic sequence.

-4	-1	-4	+4
11	26	45	68
15	19	23	
	4	4	

Work out an expression for the n th term.

Blank lined paper with horizontal ruling lines and a vertical margin line on the left side.

Answer _____

Commentary

At first glance there doesn't seem to be any worthwhile work here, but a closer look will reveal that differences have been written under the sequence, with two '4's on the second row, claiming the first mark.

1 mark

Student C

Here are the first four terms of a quadratic sequence.

11	26	45	68
----	----	----	----

Work out an expression for the n th term. [3 marks]

n	1	2	3	4	
T	11	26	45	68	
1st diff		15	19	23	
2nd diff			4	4	$\frac{4}{2} = 2$
1st					
$2n^2$	2	8	18	32	
$T - 2n^2$	9	18	27	36	
1st diff		9	9	9	
$9n$	9	18	27	36	
$2n^2 + 9n$	11	26	45	68	
$T - 2n^2 + 9n$	0	0	0	0	
$T - 2n^2 + 9n$	0	0	0	0	
Sequence is:	$2n^2 + 9n$ $2n^2 + 9n$				
Answer	<u>$2n^2 + 9n$</u>				

Commentary

The student works through the question correctly, but on the penultimate line the expression has a subtraction sign rather than an addition sign. However, the correct sign is used on the answer line, and as 'the answer line wins' full marks are awarded.

3 marks

Question 22

22 The **only** solution to $x^2 + bx + c = 0$ is $x = 5$

Work out the values of b and c .

[2 marks]

$b =$ _____ $c =$ _____

Student A

The **only** solution to $x^2 + bx + c = 0$ is $x = 5$

Work out the values of b and c .

$5^2 + b5 + c = 0$

$25 + 5b + c = 0$

$b =$ +10 -5 $c =$ 5

Commentary

The student has simply substituted 5 into the equation, which leads to a linear equation with two unknowns, which of course can't be solved.

0 marks

Student B

The **only** solution to $x^2 + bx + c = 0$ is $x = 5$

Work out the values of b and c .

$$25 + b \times 5 + \frac{c}{5} = 0$$
$$\text{if } b = -8$$
$$+ \quad -40$$
$$25 + -8 \times 5 + 15 = 0.$$
$$b = -8 \quad c = 15$$

Commentary

This student has also substituted 5 into the equation, but realising that that led nowhere has decided to let b have the value -8 (reason unknown) and see what the corresponding value of c would be. Unfortunately, this does not lead to values that would give an equation with 5 as the only solution.

0 marks

Student C

The **only** solution to $x^2 + bx + c = 0$ is $x = 5$

Work out the values of b and c .

[2 marks]

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$25 + b \times 5 + c = 0$$
$$b \times 5 + c = -25$$
$$b = \quad c =$$

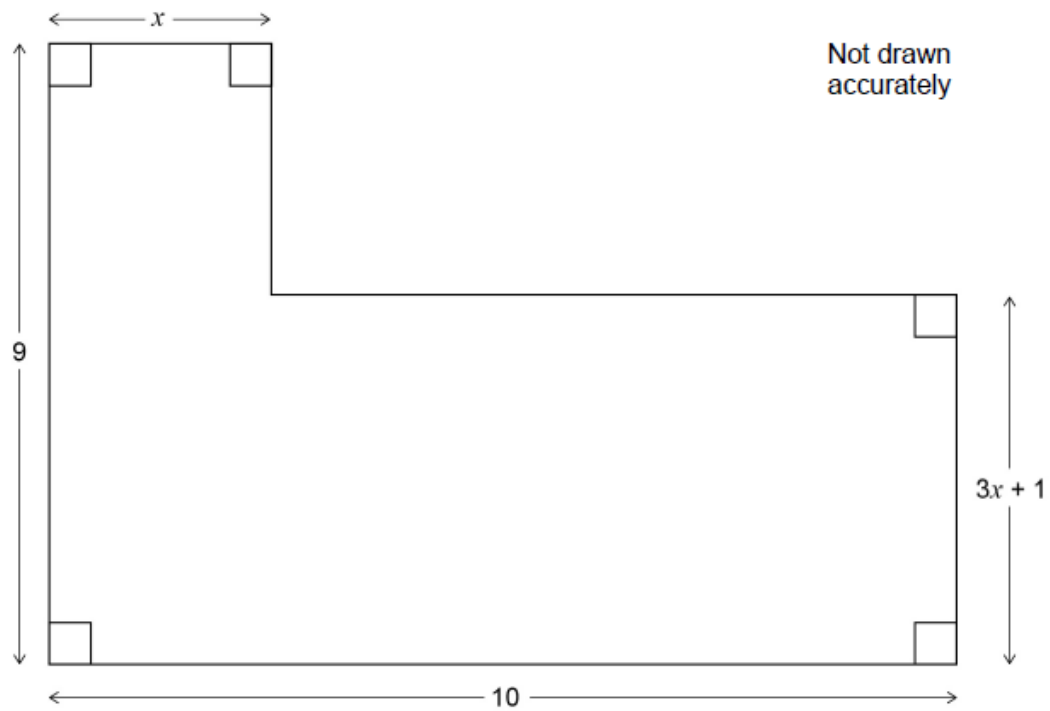
Commentary

After writing the quadratic formula and deciding that that was the wrong approach the student, like many others, has substituted 5 into the equation. As with the other examples this receives no marks.

0 marks

Question 26

- 26 Here is an L-shape.
All dimensions are in centimetres.



The area of the L-shape is 65 cm^2

Work out the value of x .

[6 marks]

Student A

The area of the L-shape is 65 cm^2

Work out the value of x .

$$\begin{aligned} \overbrace{10}^y &= 10 - x \\ w &= 9 - 3x + 1 \text{ cm} \end{aligned}$$

$$3x + 1 \text{ cm} = 9 \text{ cm}$$

$$3x = 10$$

$$\frac{10}{3} = 3.3$$

$$x + 3.3 = 10 = 6.6$$

$$x = 6.6$$

Commentary

The student has written correct expressions for the missing lengths, but to get the first mark a correct expression for any appropriate rectangle must be seen, which is not the case here.

0 marks

Student B

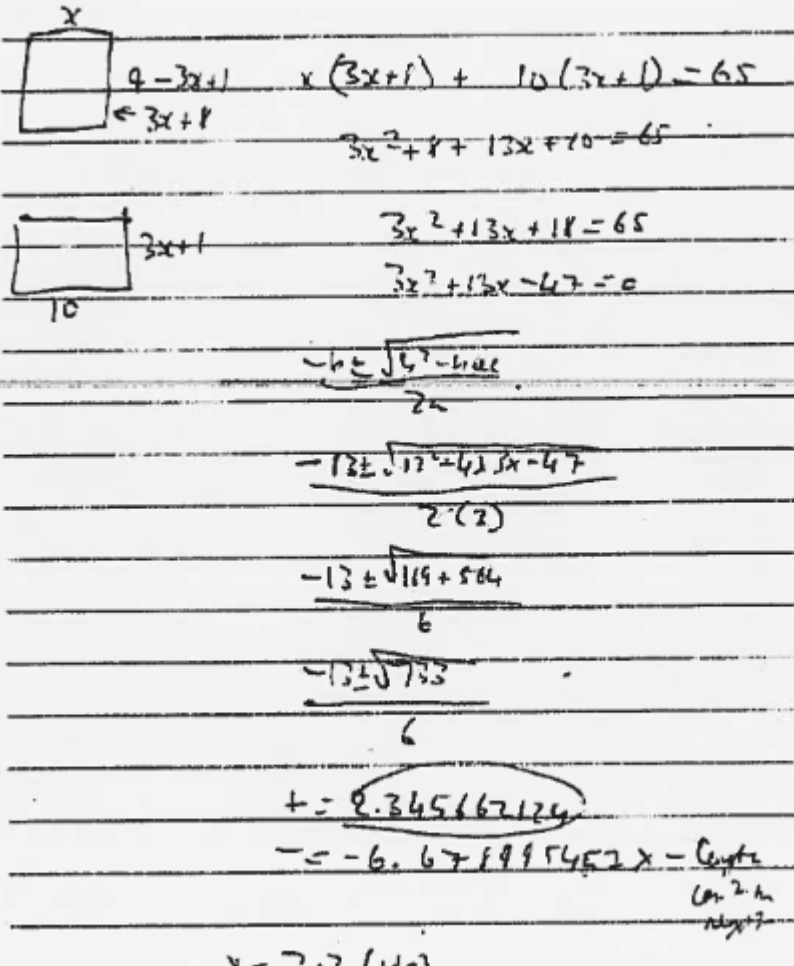
$$\begin{array}{l}
 3x + 1 \times 10 - x \\
 x \times 9 = 9x \\
 3x + 1 \times 10 - x = \\
 \quad \quad \quad 30x \\
 30x - x - 3x^2 + 10 \\
 29x - 3x^2 + 10 \\
 3x^2 - 29x + 10 \\
 \cancel{x} \quad \quad \cancel{x} \\
 \hline
 10 \times 3x + 1 = 30x + 10 \\
 9 - 3x + 1 \times x \\
 9x - 3x^2 + x \\
 \quad \quad \quad 10x - 3x^2 \\
 10x - 3x^2 + 30x + 10 \\
 30x + 10x = 40x \\
 -3x^2 + 40x + 10 \\
 -3x^2 + 40x + 10 = 65 \text{ cm}^2 \\
 -3x^2 + 40x = 55 \text{ cm}^2
 \end{array}$$

Commentary

The student did no further working and left the answer line blank. They receive the first mark for $9x$ or $29x - 3x^2 + 10$, but unfortunately don't add the two expressions to get the required area. They never end up with an appropriate quadratic equation after that, which means they can't get the fifth mark.

1 mark

Student C



$$\begin{aligned}
 & x(3x+1) + 10(3x+1) = 65 \\
 & 3x^2 + x + 13x + 10 = 65 \\
 & 3x^2 + 13x + 10 = 65 \\
 & 3x^2 + 13x - 47 = 0 \\
 & x = \frac{-13 \pm \sqrt{13^2 - 4(3)(-47)}}{2(3)} \\
 & x = \frac{-13 \pm \sqrt{169 + 564}}{6} \\
 & x = \frac{-13 \pm \sqrt{733}}{6} \\
 & x = 2.7 \text{ (1dp)}
 \end{aligned}$$

Commentary

The student has split the shape into two rectangles, and has the correct expression $10(3x+1)$ for one of them, which gets one mark, but the other expression is incorrect, as $x(3x+8)$ should be $x(8-3x)$. This denies the student the next three marks, but they do get the fifth mark for correctly using the quadratic formula with their equation. There is no ft on the last mark, so they cannot receive that for an incorrect quadratic equation.

2 marks