

2021 Assessment resources

GCSE Mathematics

Number - Higher

Answers and commentaries

The question numbers in this resource reflect the question numbers from the original papers and match the question numbers in the corresponding 2021 assessment materials.

Question 5

- 5** Write 36 as a product of prime factors.
Give your answer in index form.

[3 marks]

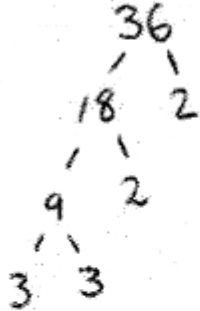
Answer _____

Student A

Write 36 as a product of prime factors.

Give your answer in index form.

[3 marks]



Answer

$$3^2 + 2^2$$

Commentary

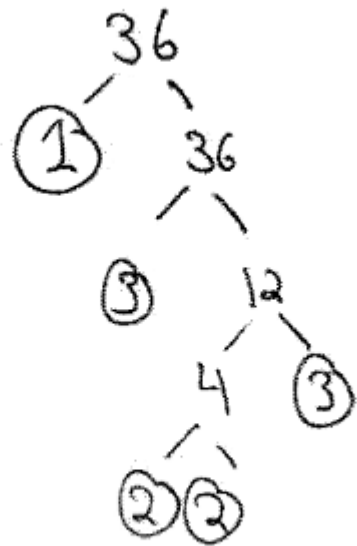
This student has used a factor tree to find the prime factors and has carried it out correctly. This scores the first two marks. Unfortunately, although the student has correctly written the values in index form, they have presented their answer as a sum rather than a product, and therefore don't get the final mark. Students must show that it is a multiplication to get the final mark.

2 marks

Student B

Write 36 as a product of prime factors.

Give your answer in index form.



Answer $1 \times 2^2 \times 3^2$

Commentary

This student has used a factor tree and found 2, 2, 3 and 3, but because of the unnecessary first step they have introduced a '1' into their working. While this is allowed for the first two marks it is not allowed for the final mark, so although the student has correctly given the 2s and 3s as a multiplication in index form the '1' loses that final mark.

2 marks

Student C

Write 36 as a product of prime factors.

Give your answer in index form.

[3 m]

3	6	2
6	12	4
9	18	6
12	24	8
15	30	10
18	36	12
21		14
24		16
27		18
30		20
33		22
36		24
		26
		28
		30
		32
		34
		36

Answer

$36 = 3 \times 12 = 6 \times 6 = 2 \times 18$

Commentary

The student has taken the unusual step of counting up in 3s, 6s and 2s to get to 36, presumably to find out how many times each number goes into 36. This leads them to write the three multiplications on the answer line. 6×6 would not get a mark, as neither number is prime, but 3×12 and 2×18 are both acceptable for the first mark (even without the multiplication signs), so the student receives one mark.

1 mark

Question 7

- 7 $\frac{3}{5}$ of a number is 162

Work out the number.

[2 marks]

Answer _____

Student A

$\frac{3}{5}$ of a number is 162

Work out the number.

$\frac{3}{5} = 162$ $\frac{100}{5} = 20 \times 3 = 60$ $\frac{100}{5} = 20 \times 3 = 60$ $\frac{3}{5} = 0.6$ [2 marks]

$100 \div 5 = 20$, $20 \times 3 = 60\%$, $100\% =$

$162 = 60\%$ $0.6 = 162$ $\frac{60}{162} = 2.8$

$2.08 = 19\%$ $1 =$

$208 = 100\%$

$\frac{3}{5} \times 208$ Answer 208

$208 \times \frac{3}{5} = 124.8$

Commentary

Rather than using the fraction this student is trying to work with percentages. They correctly show that 162 is 60% of the answer, and then try to scale this up to 100%. Their answer of 208 actually comes from dividing 162 by 60 and multiplying by 100, which is the correct method with a calculation error. As their method would have given the correct answer they received the first mark, but obviously not the second. Note that they have incorrectly written $\frac{60}{162} = 2.08$ at one point, but as their division was done in the correct order this slip is ignored.

0 marks

Student B

$\frac{3}{5}$ of a number is 162

Work out the number.

$$162 \div 3 = 54$$

$$54 \times 5 = 270$$

$$54 \times 10 = 540 \div 2 = 270$$

Answer 270

Commentary

The student does everything correctly on the first two lines, arriving at the correct answer of 270. They then do a check of their multiplication by 5 by multiplying by 10 and halving, which is a very sensible thing to do. They of course get 2 marks for the correct answer.

2 marks

Student C

$\frac{3}{5}$ of a number is 162

Work out the number.

[2 marks]

$$\begin{array}{r}
 162 \\
 \times 5 \\
 \hline
 810 \\
 31 \\
 \hline
 270 \div 5 = 54 \times 3 = 270 \quad 162
 \end{array}$$

$$\begin{array}{r}
 270 \\
 \div 3 \\
 \hline
 890 \\
 54 \times 3 \\
 \hline
 162
 \end{array}$$

Answer 270

Commentary

The student has tried two methods here, the correct one of multiplying by 5 and then dividing by 3, and the incorrect one of dividing by 5 and multiplying by 3. If they had not given an answer this would have been regarded as 'choice', and no marks would have been awarded. However, an answer has been given, so we mark the method leading to that answer. In this case, of course, the answer is correct, so the student received both marks. Had they given the answer 162, they would have received 0 marks, despite correct working being seen, as the incorrect method would have been chosen for their answer.

2 marks

Question 8

8

Work out the value of $(3^{12} \div 3^5) \div (3^2 \times 3)$

[3 marks]

Answer _____

Student A

Work out the value of $(3^{12} \div 3^5) \div (3^2 \times 3)$

$$(3^{12} \div 3^5) \div (3^2 \times 3)$$

$$3^{12} \div 3^5 = 3^7 \quad 3^2 \times 3 = 3^3$$

$$3^7 \div 3^3 = 3^4$$

Answer 3^4


Commentary

This student has not understood that when we ask for 'the value' of something we want the answer as an ordinary number. Unfortunately, this has cost the student the final mark. Had we wanted the answer in the form the student has given we would have said 'Give your answer as a power of 3'.

2 marks

Student B

Work out the value of $(3^{12} \div 3^5) \div (3^2 \times 3^6)$

1 

$$3^7 \div 3^2$$
$$= 3^5$$

Answer 3^5

Commentary

It can be seen above the given expression that the student thinks that 3 written as a power of 3 is 3^0 rather than the correct 3^1 . In this scheme there was no provision for marks being awarded for the correct interpretation of the laws of indices with incorrect indices; both expressions had to be correct for the second mark.

1 mark

Student C

Work out the value of $(3^{12} \div 3^5) \div (3^2 \times 3)$

$$3^7 \div 3^3 = 3^4$$

$$3 \times 3 \times 3 \times 3$$

Answer $3^4 / 81$

Commentary

This student presumably was not sure how to present the answer to show 'the value' as requested, so has given both numerical and index form. In this case, we were happy to award full marks, as the student has worked out the value that we wanted. In questions where we ask for the answer in a specific form we may well withhold the final mark if a student gives the answer in more than one form, but in this case we felt that withholding a mark was unwarranted.

3 marks

Question 16

Please see the mark scheme

Question 5

5 Work out $8\frac{1}{2} \div 2\frac{2}{3}$

Give your answer as a mixed number.

[4 marks]

Answer _____

Student A

Work out $8\frac{1}{2} + 2\frac{2}{3}$

Give your answer as a mixed number.

$$\frac{17}{2} \div \frac{2}{3} = \frac{17}{2} \times \frac{3}{2} = \frac{51}{4} = 12\frac{3}{4}$$

Answer

$$12\frac{3}{4}$$

Commentary

This student has completed the question in the efficient way expected by alternative method 1 in the mark scheme.

4 marks

Note that the final mark is an independent 'B' mark for correctly converting any improper fraction into a mixed number, so students who have failed to score any of the first three marks may still score the last one.

Question 6

- 6 To the nearest pound, Jon has £9
To the nearest 50p, Ellie has £6.50

Work out the maximum possible total amount of money.

[3 marks]

Answer £ _____

Student A

To the nearest pound, Jon has £9

To the nearest 50p, Ellie has £6.50

Work out the maximum possible total amount of money.

[3 marks]

$$£9 = £8.50 \text{ or } £9.50 \text{ B1}$$

$$\cancel{£0.50p} \quad £6.50 = £6. \cancel{45}25 \text{ or } £6.75$$

$$£6.75 + £9.50 = £16.25$$

m1

Answer £ 16.25 A0

Commentary

This student has failed to recognise that money is discrete, and therefore the maximum possible value is not the same as the 'upper bound' that would be applied for continuous data. It is only in the final mark that this is penalised. The first mark is awarded for any minimum amount/lower bound or any maximum amount/upper bound, and this student has given both minimum amounts and both upper bounds. The second mark is given for adding their maximum amounts, as long as they are higher than the given amounts and equal to or lower than the upper bounds. This student received that mark for using the upper bounds. The final mark is only given for the correct answer of £16.23. Note that in this question there is an SC2 for £16.26 or £16.24. This is for students who have used one or both upper bounds instead of the maximum possible values, but who have not shown enough working to qualify for the first two marks. It does not apply to this student, who had already received the first two marks.

2 marks

Student B

To the nearest pound, Jon has £9

To the nearest 50p, Ellie has £6.50

Work out the maximum possible total amount of money.

$$£9.49 + £6.54 = £16.03$$

BI

MI

Answer £

£16.03 AO

Commentary

This student has done just enough to receive two marks. They get the first mark for £9.49, which is a correct maximum value. They then get the second mark for adding their maximum amounts. £6.54 is allowed as it is more than the given £6.50 and less than or equal to the 'upper bound' of £6.50 (in this case the student has worked on rounding £6.50 to the nearest 10p). The final mark is only awarded for the correct answer, so this student does not receive it.

2 marks

Student C

To the nearest pound, Jon has £9

To the nearest 50p, Ellie has £6.50

Work out the maximum possible total amount of money.

[3 marks]

Jon has = up to £9.49 down to £8.50

Ellie has = £6.25 or £6.74

B1

$$9.49 + 6.25 = 15.74$$

M0

Answer. £

15.74

A0

Commentary

The student has given all four correct minimum and maximum possible amounts, so gets the first mark. Unfortunately, they have used the minimum value of £6.25 in their addition, and as this is obviously below the given amount they cannot qualify for the second mark.

1 mark

Question 22

22 Show that $(5\sqrt{3} - \sqrt{12})^2$ simplifies to an integer.

[3 marks]

Student A

Show that $(5\sqrt{3} - \sqrt{12})^2$ simplifies to an integer.

$$(5\sqrt{3} - \sqrt{12})^2 = 27$$

Commentary

Even though this student has come up with the correct answer of 27 they have failed to show any work whatsoever, and therefore receive no marks. Students should be aware that, although working does not have to be as formally set out as in a proof, in a 'Show that' question each step of the method should be written down.

0 marks

Student B

Show that $(5\sqrt{3} - \sqrt{12})^2$ simplifies to an integer.

~~$5\sqrt{3} - \sqrt{12}$~~

~~$5\sqrt{3} - \sqrt{12}$~~

~~$5\sqrt{3} - \sqrt{12}$~~

~~$5\sqrt{3} - \sqrt{12}$~~

$5\sqrt{3} - \sqrt{12} = 3\sqrt{3}$

$(3\sqrt{3})^2 = 27$

$(5\sqrt{3} - \sqrt{12})^2 = 27$

Commentary

There is only one thing missing here, but unfortunately it is vital and comes early in the working, therefore losing all marks. The first line of working is correct, but insufficient for a 'Show that' question. The student must show that $\sqrt{12} = 2\sqrt{3}$ before subtracting it from $5\sqrt{3}$. This may seem harsh, but of course this was on a calculator paper, and therefore students could easily get the correct values from their calculators. In a 'Show that' question it is usual for the scheme to be dependent, so that as soon as one mark is lost all further marks are automatically lost.

0 marks

Student C

Show that $(5\sqrt{3} - \sqrt{12})^2$ simplifies to an integer.

$$\begin{aligned} & (5\sqrt{3} - 2\sqrt{3})^2 \\ & \quad \downarrow \\ & (3\sqrt{3})^2 \\ & \quad \downarrow \\ & 3\sqrt{3} \times 3\sqrt{3} = 9 \times 3\sqrt{3} \times 3\sqrt{3} \times 3 \\ & \quad \downarrow \\ & 9 \times 27 \times 3 \\ & \quad \downarrow \\ & \underline{729} \end{aligned}$$

Commentary

By its position in the bracket it is clear that the student has changed $\sqrt{12}$ to $2\sqrt{3}$ for the first mark, and they have successfully subtracted for the second mark. Unfortunately, they seem to have treated the expansion of the bracket as if it had two terms, and have therefore lost the final mark.

2 marks

Question 15

15

Meal Deal

Choose one sandwich, one drink and one snack

There are

7 different sandwiches

5 different drinks

and

3 different snacks.

15 (a) How many different Meal Deal combinations are there?

[2 marks]

Answer _____

Student A

How many different Meal Deal combinations are there?

105 $7 \times 5 \times 3 = 105$

Answer _____

Commentary

Even though the answer line has been left blank, the correct answer is clearly shown coming from correct working, so full marks can be awarded.

2 marks

Student B

How many different Meal Deal combinations are there?

7 different sandwiches

Answer 7

Commentary

This student has simply given the number of different sandwiches, which scores no marks. They should perhaps have realised that two marks would not have been awarded for simply copying down one number.

0 marks

Student C

How many different Meal Deal combinations are there?

Answer

105

Commentary

This student has shown no working, but as the answer is correct they score both marks. If working is required this will be indicated in the accuracy mark section of the scheme.

2 marks

Question 11

11

Work out

cube root of 512 : reciprocal of 0.4

Give your answer in the form $n : 1$

[3 marks]

Answer _____ : _____

Student A

Work out

cube root of 512 : reciprocal of 0.4

Give your answer in the form $n : 1$

51

$$\sqrt[3]{512} = 8$$

$$\text{reciprocal of } 0.4 = \frac{1}{0.4}$$

$$8 \div \frac{1}{0.4} = 20$$

Answer 20 : 1

Commentary

This scheme starts with an independent B mark for 8, which this student receives. They simply turn 0.4 into a fraction, so don't receive the second mark. They do correctly turn their values of 8 and 0.4 into a ratio in the form $n : 1$, but in this question the follow through on the accuracy (A) mark is only for an incorrect 8, not an incorrect 2.5. This is to avoid students who have no understanding of reciprocals receiving two marks for, for example, $8 : 4 = 2 : 1$

1 mark

Student B

Work out

S3

cube root of 512 : reciprocal of 0.4

Give your answer in the form $n : 1$

$$\sqrt[3]{512} = 67.88 : \frac{5}{2}$$

Answer 67.88 : 5/2

Commentary

This student thinks that you divide by 3 to get the square root, so loses the first mark. They correctly work out the reciprocal of 0.4, so get the second mark and would get the third for an answer of $27.152 : 1$, which they should be able to do easily on a calculator paper, but they have either failed to read the last line of the question or do not know how to convert their ratio.

1 mark

Student C

Work out

cube root of 512 : reciprocal of 0.4

Give your answer in the form $n : 1$

S8

$$1 \div 0.4 = 2.5$$

$$512 \times 2.5 = 1280$$

Answer 1280 : 1

Commentary

This student has done nothing with the 512, so does not get the first mark. They have correctly worked out the reciprocal of 0.4, so do get the second mark. They have tried to convert the ratio into the form $n : 1$, but have multiplied 512 by 2.5 instead of dividing. An answer of 204.8 : 1 would have seen them score B0M1A1ft.

1 mark

Question 15

- 15** When you earn money you pay income tax.
The amount you pay depends on how much you earn that year.
You pay
- | | | |
|-----|--------------|--------------------|
| 0% | on the first | £12 500 you earn |
| 20% | on the next | £37 500 you earn |
| 40% | on the next | £112 500 you earn. |

One year, Kim paid £9260 income tax.

Work out how much she earned that year.

[4 marks]

Answer £ _____

Student A

You pay

0% on the first £12 500 you earn
7500 20% on the next £37 500 you earn 50000
45000 40% on the next £112 500 you earn. 162500

One year, Kim paid £9260 income tax.

Work out how much she earned that year.

50000 - 162500

Between 7500 and 45000

$37500 \times 0.25 = 9375$

Answer £ 37500

Commentary

7500 is shown to the left of the second indent and in the working, which scores the first mark. The student seems to have missed the point of the question from there and does not mention the pivotal amount of 9260, so cannot score any further marks.

1 mark

Student B

One year, Kim paid £9260 income tax.

Work out how much she earned that year.

[4 marks]

Income tax £9260

first 12500 nothing
 $\frac{20\%}{100} \quad 37500 = \underline{\underline{£7500}}$

$\frac{40\%}{100} \quad 112500 = 45000$
 $9260 - 7500 = 1760$
 $9260 + 1760 = 11020 \div 100 = 110.2$

Answer £ _____

$\frac{110.2}{100} \quad 9260 = 10204.52$

Commentary

This student scores the first mark for 7500 and correctly subtracts that amount from 9260 to get 1760. Unfortunately, they have to divide that amount by 0.4 to get the second mark, but they get lost from that point.

1 mark

Student C

Work out how much she earned that year.

$$9260 \times 1.2 = 11112$$

$$9260 \times 1.2 = 11112 \times$$

$$110,000 \div 20 = 5500$$

She paid 40% of her income.

$$200,000 \div 40 = 5000 \text{ more}$$

$$300,000 \div 40 = 7500 \text{ more}$$

$$360,000 \div 40 = 9000 \text{ close}$$

Answer £ 370,400.

$$365,500 \div 40$$

$$365,000 \div 40 = 9125$$

$$368,000 \div 40 = 9200$$

$$370,000 \div 40 = 9250$$

$$371,800 \div 40$$

Commentary

Although this student has mentioned the value 7500, it comes from clearly incorrect working of $300\,000 \div 40$ and is therefore not awarded the first mark.

0 marks

Question 6

6 Show that 268 can be written as the sum of a power of 3 and a square number.

[2 marks]

Answer _____

Student A

6 Show that 268 can be written as the sum of a power of 3 and a square number.

[2 marks]

$$2^3 \times 5.8^2 = 268$$

$$2^3 = 8 \quad 5.8^2 = 33.64$$

Answer $2^3 \times 5.8^2$ _____

Commentary

The student has misunderstood 'a power of 3' as being a number cubed rather than 3 raised to a power, and therefore has done no work worthy of a mark.

0 marks

Student B

Show that 268 can be written as the sum of a power of 3 and a square number.

[2 marks]

$$\sqrt{268} = 2\sqrt{67}$$

$$\begin{array}{r} 1268 \\ -1256 \\ \hline \sqrt{1256} = \end{array}$$

$$268$$

$$16^3$$

$$\begin{array}{r} 268 \\ -225 \\ \hline 43 \end{array}$$

$$\sqrt{225} = 15$$

Answer $16^2 + 12$

Commentary

The student has done some work apparently centred on 15^2 and then decided to use 16^2 and simply add the extra amount required. This scores no marks.

0 marks

Student C

Show that 268 can be written as the sum of a power of 3 and a square number.

[2 marks]

$$268$$

$$7^2 + 6^3 = 265 \times$$

$$12^2 + 5^3 = 269 \times$$

$$\sqrt[3]{268} = 16.37$$

$$\sqrt{268}$$

Answer _____

Commentary

This student thinks that the sum of a square number and a cube number is required and therefore receives no marks.

0 marks

Question 27

27 Prove algebraically that $2.\dot{7}5$ converts to the fraction $\frac{124}{45}$ **[3 marks]**

Student A

Prove algebraically that $2.\dot{7}5$ converts to the fraction $\frac{124}{45}$

[3 marks]

$$x = 2.\overset{4}{7}5555$$

M 1

$$100x = 275.5555$$

$$10x = 27.555$$

$$100x - 10x = 275.5555 - 27.5 = 248$$

$$90x = 248$$

Mdep 0

$$\frac{248}{90}$$

$$\frac{124}{45}$$

Commentary

This student has lost two marks by not setting out their work appropriately. When students are asked to 'prove' something they must set out their work formally and sequentially. In this case, it was imperative that students set up an equation linking the numerical and algebraic expressions, but unfortunately this student has not done so, so they are awarded only the first mark.

1 mark

Student B

Prove algebraically that $2.7\dot{5}$ converts to the fraction $\frac{124}{45}$

[3 marks]

$$2.7\dot{5} = \frac{124}{45}$$

M 0

$$2.7\dot{5} \times 45 = 124$$

$$x = 2.7\dot{5} \quad 45x = 124$$

$$x = \frac{124}{45} = 2.7\dot{5}$$

~~124/45~~

Commentary

This student has done as many do when asked to prove that two things are equal; set them up as an equation. Unfortunately, this is not the way to conduct a proof. Students must start with the first expression and show step-by-step how it can be turned into the second one.

0 marks

Student C

Prove algebraically that $2.7\dot{5}$ converts to the fraction $\frac{124}{45}$

$$2.7\dot{5} = x$$

$$27.\dot{5} = 10x$$

$$275.\dot{5} = 100x$$

$$100x - 10x = 275.\dot{5} - 27.\dot{5}$$
$$= 248$$

$$= \frac{248}{90} = \frac{124}{45}$$

Commentary

This student was doing everything right until they omitted '90x' from the left-hand side of the equation. This broke the string of equations needed for a proof, so they lost the final mark.

2 marks