Cambridge Assessment International Education

Cambridge O Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

3764988855

ADDITIONAL MATHEMATICS **4037/12**

Paper 1 **May/June 2023**

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

DC (ST) 326646

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $\quad u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $\quad u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

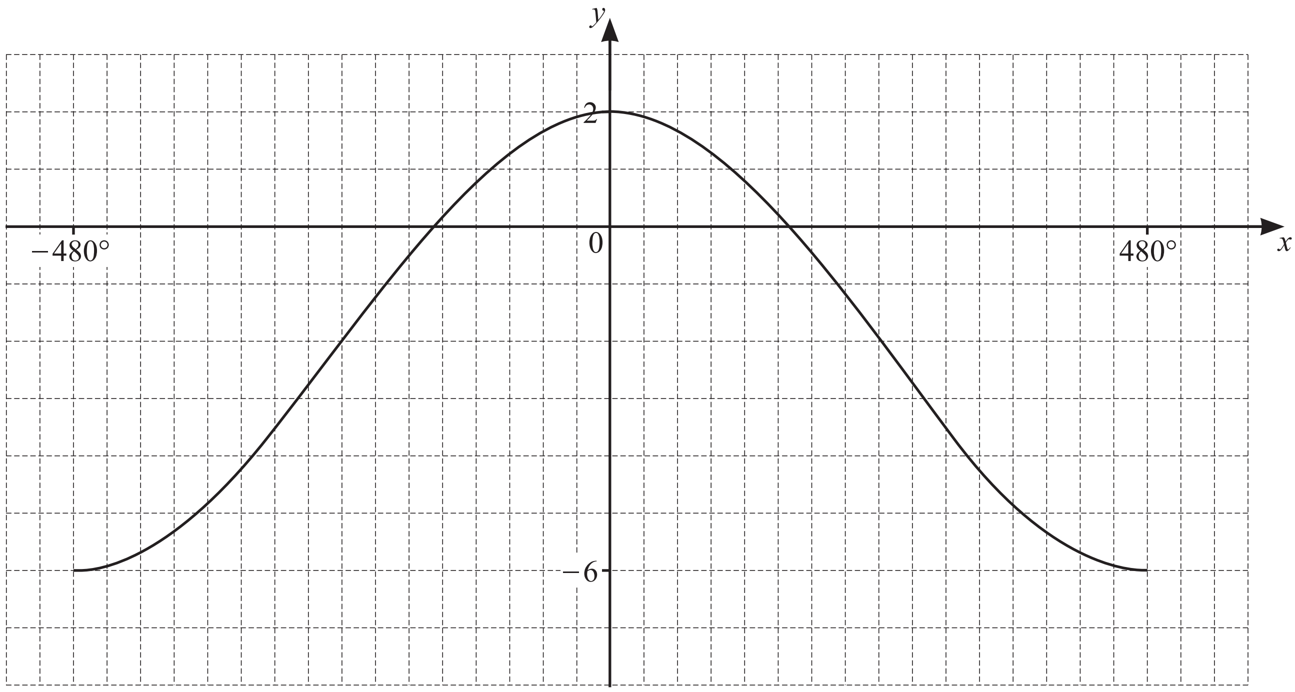
Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 The diagram shows the graph of $y = a\cos bx + c$. Find the values of the constants a, b and c. [3]



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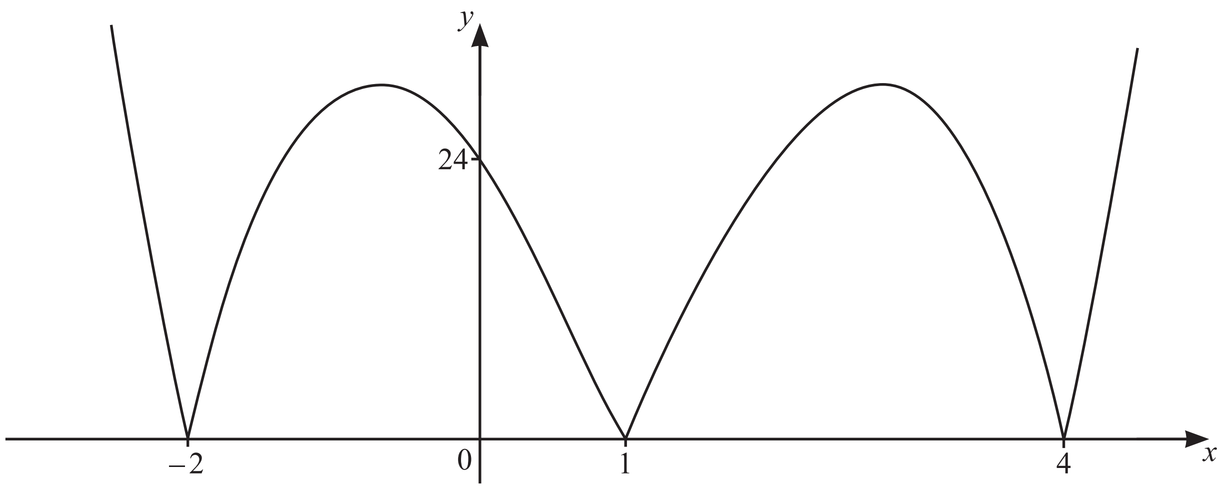
2 DO NOT USE A CALCULATOR IN THIS QUESTION.

Solve the equation $\left(2+\sqrt{5}\right)x^2 = 4x + 3\left(2-\sqrt{5}\right)$, giving your answers in the form $a+b\sqrt{5}$ where a and b are integers. [5]

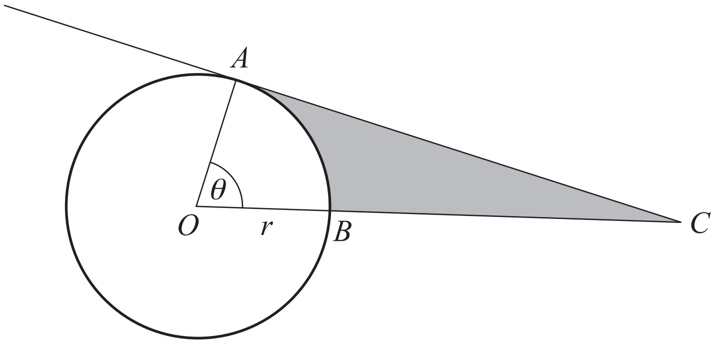
3 **(a)**



The diagram shows the graph of $y = |f(x)|$, where $f(x)$ is a cubic polynomial. Find, in factorised form, the possible expressions for $f(x)$. [3]

(b) Solve the inequality $|5x - 2| \leqslant |4x + 1|$. [4]

4 In this question all lengths are in centimetres and all angles are in radians.



The diagram shows a circle with centre O and radius r. The points A and B lie on the circumference of the circle such that the angle AOB is θ and the length of the minor arc AB is 12. The area of the minor sector AOB is $57.6\,\text{cm}^2$. The point C lies on the tangent to the circle at A such that OBC is a straight line.

(a) Find the values of r and θ. [4]

(b) Find the area of the shaded region. Give your answer correct to 1 decimal place. [3]

5 **(a)** Find the exact solutions of the equation $6p^{\frac{1}{3}} - 5p^{-\frac{1}{3}} - 13 = 0$. [4]

(b) Solve the equation $2\lg(2x+5) - \lg(x+2) = 1$, giving your answers in exact form. [6]

6 **(a)** Given that $\cot^2\theta = \dfrac{1}{y+2}$ and $\sec\theta = x-4$, find y in terms of x. [2]

(b) Solve the equation $\sqrt{3}\,\mathrm{cosec}\left(2\phi + \dfrac{3\pi}{4}\right) = 2$, for $-\pi < \phi < \pi$, giving your answers in terms of π. [5]

7 **(a)** Find the number of ways in which 14 people can be put into 4 groups containing 2, 3, 4 and 5 people. [3]

(b) 6-digit numbers are to be formed using the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. Each digit may be used only once in any 6-digit number. A 6-digit number must not start with 0. Find how many 6-digit numbers can be formed if

(i) there are no further restrictions [1]

(ii) the 6-digit number is divisible by 10 [1]

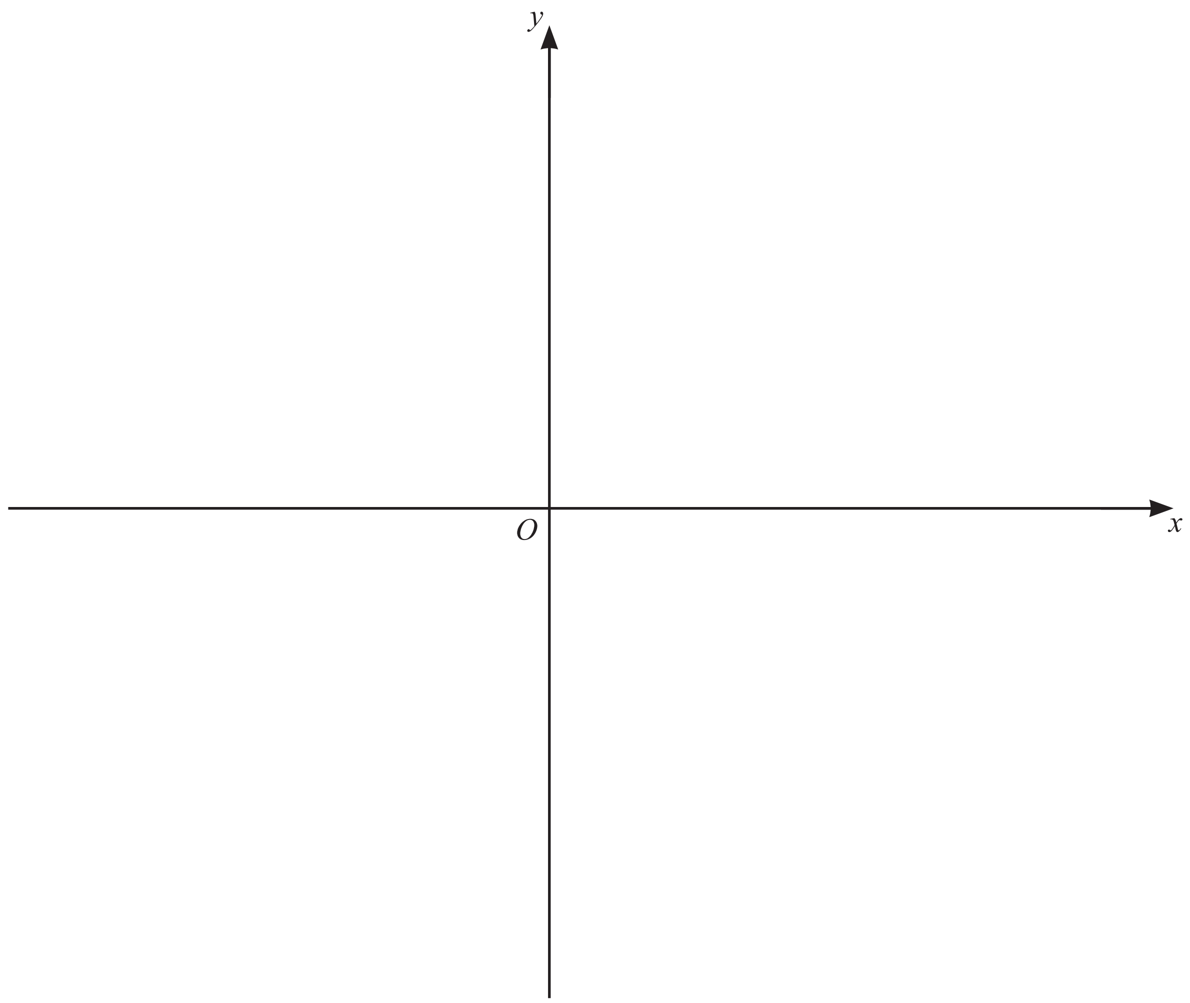
(iii) the 6-digit number is greater than 500 000 and even. [3]

8 It is given that $f(x) = 2\ln(3x-4)$ for $x > a$.

(a) Write down the least possible value of a. [1]

(b) Write down the range of f. [1]

(c) It is given that the equation $f(x) = f^{-1}(x)$ has two solutions. (You do not need to solve this equation). Using your answer to **part (a)**, sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the axes below, stating the coordinates of the points where the graphs meet the axes. [4]

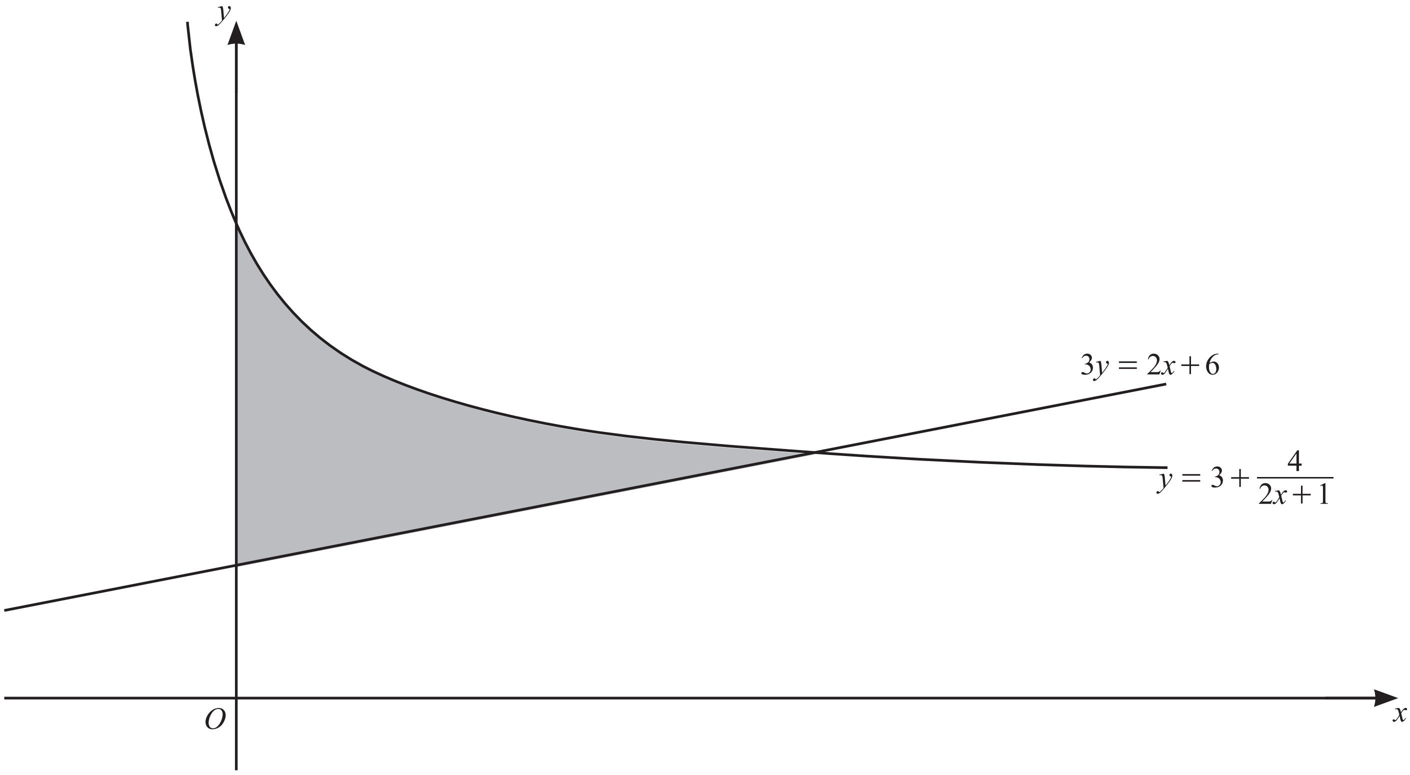


It is given that $g(x) = 2x - 3$ for $x \geqslant 3$.

(d) **(i)** Find an expression for $g(g(x))$. [1]

(ii) Hence solve the equation $fg(g(x)) = 4$ giving your answer in exact form. [3]

9



The diagram shows part of the curve $y = 3 + \frac{4}{2x+1}$ and the straight line $3y = 2x + 6$. Find the area of the shaded region, giving your answer in exact form. [10]

Continuation of working space for Question 9.

10 **(a)** The first three terms of an arithmetic progression are $(2x+1)$, $4(2x+1)$ and $7(2x+1)$, where $x \neq -\frac{1}{2}$.

(i) Show that the sum to n terms can be written in the form $\frac{n}{2}(2x+1)(An+B)$, where A and B are integers to be found. [2]

(ii) Given that the sum to n terms is $(54n+37)(2x+1)$, find the value of n. [2]

(iii) Given also that the sum to n terms in **part (ii)** is equal to 1017.5, find the value of x. [2]

(b) The first three terms of a geometric progression are $(2y+1)$, $3(2y+1)^2$ and $9(2y+1)^3$, where $y \neq -\frac{1}{2}$.

Given that the nth term of the progression is equal to 4 times the $(n+2)$th term, find the possible values of y, giving your answers as fractions. [4]

(c) The first three terms of a different geometric progression are $\sin\theta$, $2\sin^3\theta$ and $4\sin^5\theta$, for $0 < \theta < \frac{\pi}{2}$. Find the values of θ for which the progression has a sum to infinity. [3]

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