

Please check the examination details below before entering your candidate information

Candidate surname

Other names

Centre Number

Candidate Number

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## Pearson Edexcel International GCSE

Time 2 hours

Paper  
reference

**4PM1/02R**

### Further Pure Mathematics PAPER 2R



Calculators may be used.

Total Marks

### Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided  
– *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.  
Anything you write on the formulae page will gain NO credit.

### Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets  
– *use this as a guide as to how much time to spend on each question.*

### Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

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Pearson

## International GCSE in Further Pure Mathematics Formulae sheet

### Mensuration

Surface area of sphere =  $4\pi r^2$

Curved surface area of cone =  $\pi r \times$  slant height

Volume of sphere =  $\frac{4}{3}\pi r^3$

### Series

#### Arithmetic series

Sum to  $n$  terms,  $S_n = \frac{n}{2}[2a + (n - 1)d]$

#### Geometric series

Sum to  $n$  terms,  $S_n = \frac{a(1 - r^n)}{(1 - r)}$

Sum to infinity,  $S_\infty = \frac{a}{1 - r} \quad |r| < 1$

#### Binomial series

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad \text{for } |x| < 1, n \in \mathbb{Q}$$

### Calculus

#### Quotient rule (differentiation)

$$\frac{d}{dx} \left( \frac{f(x)}{g(x)} \right) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

### Trigonometry

#### Cosine rule

In triangle  $ABC$ :  $a^2 = b^2 + c^2 - 2bc \cos A$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

### Logarithms

$$\log_a x = \frac{\log_b x}{\log_b a}$$

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**Answer all ELEVEN questions.**

**Write your answers in the spaces provided.**

**You must write down all the stages in your working.**

- 1 (a) Expand  $\left(1 + \frac{x}{4}\right)^8$  in ascending powers of  $x$  up to and including the term in  $x^3$ .  
Give each coefficient in its simplest terms. (3)
  
- (b) Use your expansion with a suitable value of  $x$  to obtain an approximation, to 4 decimal places, of  $(1.035)^8$ . (3)

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(Total for Question 1 is 6 marks)

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2 Find the set of values of  $x$  for which

(a)  $3x - 8 < 5x + 3$  (1)

(b)  $4x^2 - 7x + 1 > 6 - 2x^2$  (4)

(c) **both**  $3x - 8 < 5x + 3$  **and**  $4x^2 - 7x + 1 > 6 - 2x^2$  (1)

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**Question 2 continued**

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**(Total for Question 2 is 6 marks)**



3 Given that  $y = e^{3x} \sin 2x$

show that  $13y + \frac{d^2y}{dx^2} = 6 \frac{dy}{dx}$

(8)

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**Question 3 continued**

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**(Total for Question 3 is 8 marks)**



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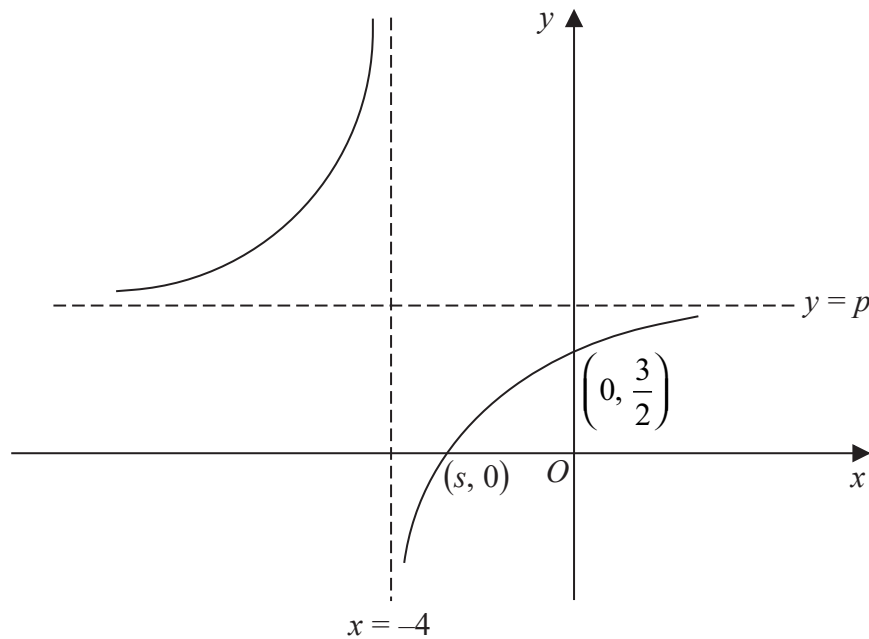


Diagram NOT  
accurately drawn

**Figure 1**

Figure 1 shows part of the curve  $C$  with equation

$$y = \frac{2x + q}{x + r} \quad x \neq -r$$

where  $q$  and  $r$  are integers.

The asymptote to  $C$  that is parallel to the  $y$ -axis has equation  $x = -4$

The asymptote to  $C$  that is parallel to the  $x$ -axis has equation  $y = p$

(a) Write down

(i) the value of  $p$

(ii) the value of  $r$

(2)

Given that  $C$  crosses the  $y$ -axis at the point with coordinates  $\left(0, \frac{3}{2}\right)$

(b) find the value of  $q$

(2)

Given that  $C$  crosses the  $x$ -axis at the point with coordinates  $(s, 0)$

(c) find the value of  $s$

(2)

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**Question 4 continued**

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**(Total for Question 4 is 6 marks)**



5 The line  $l$  with gradient  $-\frac{1}{12}$  passes through the points  $A$  and  $B$  with coordinates  $(p, 10)$  and  $(123, 0)$  respectively.

(a) Show that  $p = 3$

(2)

(b) Find an equation for  $l$  in the form  $rx + sy + t = 0$  where  $r, s$  and  $t$  are integers.

(2)

The line  $k$  is perpendicular to  $l$  and passes through the point  $A$ .

(c) Find an equation for  $k$  in the form  $y = mx + c$

(3)

Line  $k$  intersects the  $x$ -axis at the point  $C$ .

(d) Find the exact area of triangle  $ABC$ .

(4)

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**Question 5 continued**

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**Question 5 continued**

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**Question 5 continued**

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**(Total for Question 5 is 11 marks)**



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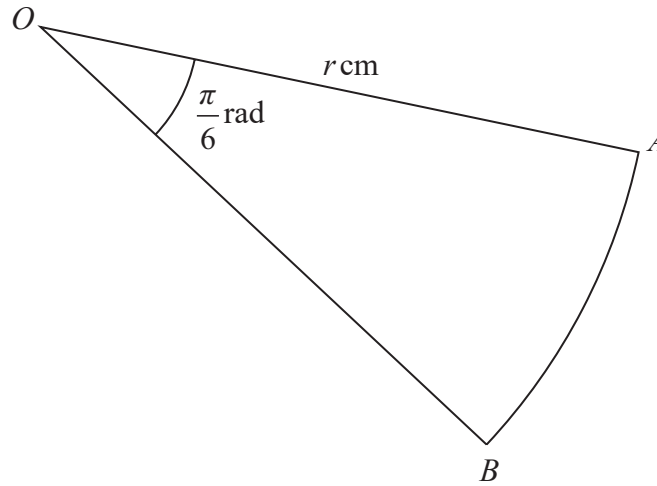


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Figure 2

Figure 2 shows the sector  $OAB$  of a circle with centre  $O$  and radius  $r$  cm.

$$\angle AOB = \frac{\pi}{6} \text{ radians} \quad OA = OB = r \text{ cm}$$

The area of the sector is increasing in such a way that the size of  $\angle AOB$  remains constant, and the lengths  $OA$  and  $OB$  are both increasing at a constant rate of  $0.2 \text{ cm/s}$

Find the exact rate of change, in  $\text{cm}^2/\text{s}$ , of the area of the sector when the length of

arc  $AB$  is  $\frac{5\pi}{2}$

(6)

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**Question 6 continued**

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**(Total for Question 6 is 6 marks)**



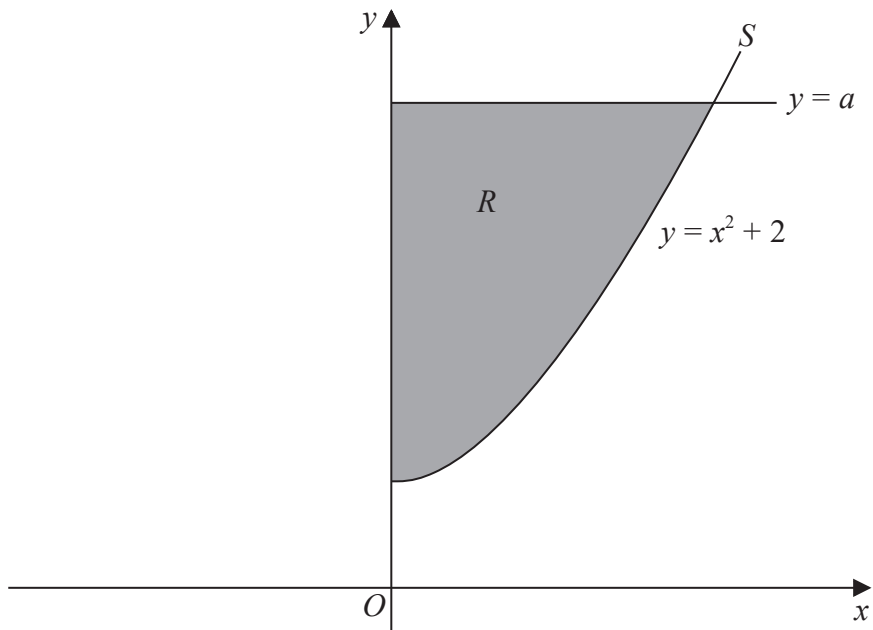


Diagram NOT accurately drawn

Figure 3

Figure 3 shows part of the curve  $S$  with equation  $y = x^2 + 2$

The finite region  $R$ , shown shaded in Figure 3, is bounded by  $S$ , the  $y$ -axis and the line with equation  $y = a$  where  $a > 2$

The region  $R$  is rotated through  $360^\circ$  about the  $y$ -axis to generate a solid with volume  $18\pi$

Use algebraic integration to find the value of  $a$

(8)

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**Question 7 continued**

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**Question 7 continued**

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**Question 7 continued**

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**(Total for Question 7 is 8 marks)**



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8 The quadratic equation  $3x^2 - kx - 1 = 0$ , where  $k$  is a positive integer, has roots  $\alpha$  and  $\beta$

(a) Show that  $\alpha^2 + \beta^2 = \frac{k^2 + 6}{9}$  (3)

Given that  $\alpha^4 + \beta^4 = \frac{466}{81}$

(b) find the value of  $k$  (5)

(c) Hence form an equation, with integer coefficients, which has roots

$$\frac{\alpha^3 + \beta}{\beta} \text{ and } \frac{\beta^3 + \alpha}{\alpha}$$

(6)



**Question 8 continued**

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**Question 8 continued**

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**Question 8 continued**

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**(Total for Question 8 is 14 marks)**



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9 A geometric series  $G$  has common ratio  $r$  where  $r > 0$

The third term of  $G$  is  $\frac{27}{2}$  and the sum of the first three terms of  $G$  is  $\frac{57}{2}$

Given that the sum to  $n$  terms of  $G$  is  $S_n$

(a) show that  $S_n = \sum_{j=1}^n 4\left(\frac{3}{2}\right)^j$  (8)

Given that  $S_k > 50\,000$

(b) show that the least value of  $k$  is given by

$$k > \frac{\lg\left(\frac{12503}{3}\right)}{\lg\left(\frac{3}{2}\right)}$$

(3)

(c) Hence find the least value of  $k$  (1)

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**Question 9 continued**

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**Question 9 continued**

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**Question 9 continued**

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**(Total for Question 9 is 12 marks)**



10 (a) Show that  $\frac{9^{3y}}{243} = 3^{(6y-5)}$

(4)

(b) Solve the simultaneous equations

$$\frac{9^{3y}}{243} = 27^{(x-2)}$$

$$\log_{10} \sqrt{6xy} = \log_4 2$$

(9)

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**Question 10 continued**

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**Question 10 continued**

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**Question 10 continued**

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**(Total for Question 10 is 13 marks)**



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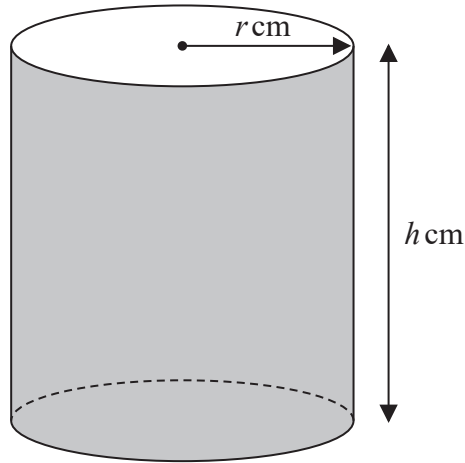


Diagram **NOT** accurately drawn

**Figure 4**

Figure 4 shows an open container in the shape of a cylinder with radius  $r$  cm and height  $h$  cm.

Given that the total surface area of the container is  $625\pi$  cm<sup>2</sup>

(a) show that

$$h = \frac{625 - r^2}{2r} \tag{3}$$

The volume of the container is  $V$  cm<sup>3</sup>

Given that  $r$  can vary,

(b) use calculus to find the value, to 3 significant figures, of  $r$  for which  $V$  is a maximum.

Justify that this value of  $r$  gives a maximum value of  $V$  (6)

(c) For the value of  $r$  found in part (b), find the corresponding value, to 3 significant figures, of  $h$  (1)

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**Question 11 continued**

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**Question 11 continued**

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**Question 11 continued**

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**Question 11 continued**

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**(Total for Question 11 is 10 marks)**

**TOTAL FOR PAPER IS 100 MARKS**

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