OCR
Oxford Cambridge and RSA

Thursday 19 June 2025 – Afternoon

A Level Mathematics A

H240/03 Pure Mathematics and Mechanics

Time allowed: 2 hours

QP

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **16** pages.

ADVICE
- Read each question carefully before you start your answer.

Formulae
A Level Mathematics A (H240)

Arithmetic series

$$S_n = \tfrac{1}{2}n(a+l) = \tfrac{1}{2}n\{2a+(n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$$

where ${}^nC_r = {}_nC_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1, n \in \mathbb{R})$$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

Quotient rule $y = \dfrac{u}{v}$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\dfrac{\mathrm{d}u}{\mathrm{d}x} - u\dfrac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Integration

$$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$$

$$\int \mathrm{f}'(x)(\mathrm{f}(x))^n\,\mathrm{d}x = \frac{1}{n+1}(\mathrm{f}(x))^{n+1} + c$$

Integration by parts $\displaystyle\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \tfrac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k + \tfrac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\int_a^b y\,\mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Standard deviation

$\sqrt{\dfrac{\sum(x-\bar{x})^2}{n}} = \sqrt{\dfrac{\sum x^2}{n} - \bar{x}^2}$ **or** $\sqrt{\dfrac{\sum f(x-\bar{x})^2}{\sum f}} = \sqrt{\dfrac{\sum fx^2}{\sum f} - \bar{x}^2}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = x) = \dbinom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\overline{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\overline{X} - \mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that $\mathrm{P}(Z \leqslant z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line	Motion in two dimensions
$v = u + at$	$\mathbf{v} = \mathbf{u} + \mathbf{a}t$
$s = ut + \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$
$s = \frac{1}{2}(u+v)t$	$\mathbf{s} = \frac{1}{2}(\mathbf{u}+\mathbf{v})t$
$v^2 = u^2 + 2as$	
$s = vt - \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

Section A
Pure Mathematics

1 **(a)** Express $3x^2 - 12x + 17$ in the form $a(x-b)^2 + c$ where a, b and c are constants. **[3]**

(b) State the coordinates of the minimum point of the curve $y = 3x^2 - 12x + 17$. **[1]**

(c) State the equation of the normal to the curve $y = 3x^2 - 12x + 17$ at its minimum point. **[1]**

2 **(a)** The curve $y = \frac{1}{x^3}$ is translated by 4 units in the positive x-direction.

Write down the equation of the curve after it has been translated. **[2]**

(b) The curve $y = 5x^2$ is stretched parallel to the y-axis with scale factor 4.

The point on the curve $y = 5x^2$ with x-coordinate 3 is transformed to the point P.

Write down the coordinates of P. **[2]**

3 **(a)** Sketch the graph of $y = |2x - 5|$ on the grid provided in the Printed Answer Booklet. **[2]**

(b) Solve the inequality $|2x - 5| < 1$. **[2]**

(c) **In this question you must show detailed reasoning.**

Hence find **all** the possible integer values N that satisfy the inequality

$|2e^{0.1N} - 5| < 1$. **[3]**

4 Functions f and g are defined for all real values of x by

$\mathrm{f}(x) = \dfrac{x-k}{2}$ and $\mathrm{g}(x) = x^2 + kx + 5$, where k is a constant.

You are given that the equation $\mathrm{f}^{-1}\mathrm{g}(x) = 4 - 2kx$ has real distinct roots.

(a) Show that k satisfies the inequality $2k^2 - k - 6 > 0$. **[5]**

(b) **In this question you must show detailed reasoning.**

Hence find the set of values of k. Give your answer in set notation. **[3]**

5 **(a)** Use the substitution $u = \cos x$ to show that

$$\int \frac{1+\cos x}{\sin x}\,dx = \ln(1-\cos x) + c.$$ **[4]**

Fig. 1

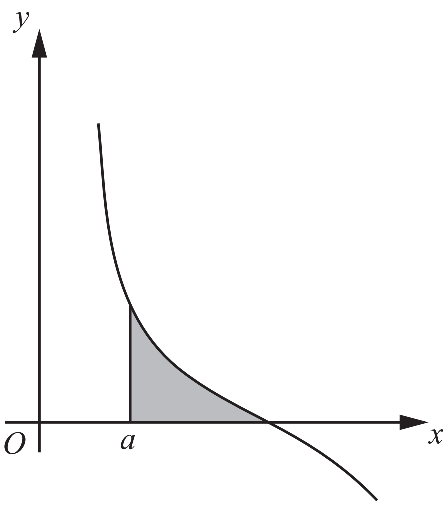


Fig. 1 shows part of the curve $y = \dfrac{1+\cos x}{\sin x}$.

The shaded region is bounded by the curve, the x-axis, and the line $x = a$.

You are given that the area of the shaded region is a square units.

(b) Show that the value of a satisfies the equation $a - \cos^{-1}(1-2e^{-a}) = 0$. **[4]**

(c) Show by calculation that the value of a lies between 1.1 and 1.2. **[2]**

(d) Use the iterative formula

$$a_{n+1} = \cos^{-1}(1-2e^{-a_n}),$$

with starting value $a_1 = 1.2$, to find the value of a correct to **2** decimal places. Show the result of each step of the iterative process. **[2]**

Fig. 2

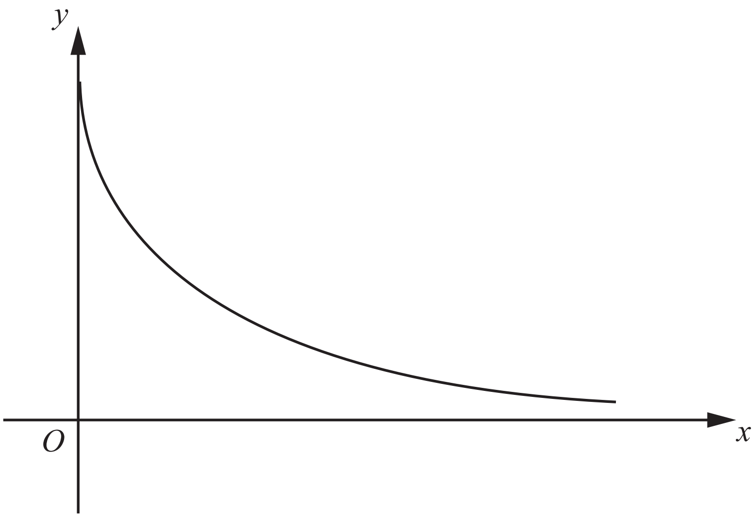


Fig. 2 shows the curve $y = \cos^{-1}(1 - 2e^{-x})$.

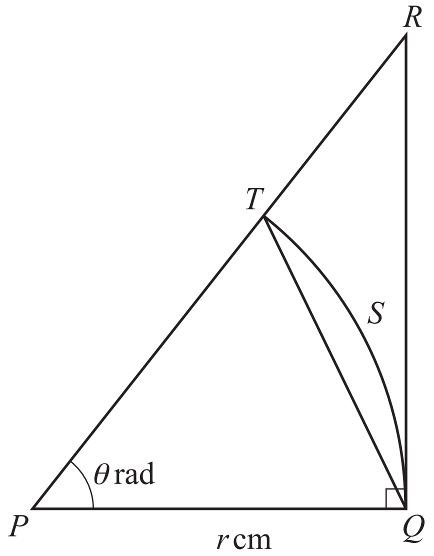
(e) Explain why the gradient of the curve $y = \cos^{-1}(1 - 2e^{-x})$ at the point where $x = a$, where a is the value found in part **(d)**, lies in the interval $(-1, 0)$. **[2]**

6 The compound angle formulae for $\sin(A+B)$ and $\sin(A-B)$ are

$\sin(A+B) = \sin A\cos B + \cos A\sin B$ and
$\sin(A-B) = \sin A\cos B - \cos A\sin B$.

(a) By letting $C = A + B$ and $D = A - B$, show that

$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$. **[1]**



The diagram shows a right-angled triangle PQR with $PQ = r$ cm. The angle QPR is θ radians. The diagram also shows the sector $PQST$ of a circle with centre P and radius r cm. The line segment QT is a chord of the sector $PQST$.

(b) By considering the areas of triangle PQT, sector $PQST$ and triangle PQR, show that

$1 < \frac{\theta}{\sin\theta} < \frac{1}{\cos\theta}$. **[4]**

(c) **(i)** Hence write down a similar inequality interval for the expression $\frac{\sin\theta}{\theta}$. **[1]**

(ii) Hence state $\lim_{\theta \to 0} \frac{\sin\theta}{\theta}$. **[1]**

(d) Using parts **(a)** and **(c)(ii)**, show from first principles that the derivative of $\sin x$ is $\cos x$, where x is measured in radians. **[4]**

A student attempts to use the result regarding the derivative of $\sin x$ to find the derivative of $\cos x$. The student's attempt is shown below.

Let	$y = \cos x$, where x is measured in radians.
	$y = \sin\left(\frac{\pi}{2} - x\right)$
so	$\frac{\mathrm{d}y}{\mathrm{d}x} = \cos\left(\frac{\pi}{2} - x\right)$
but	$\sin x \equiv \cos\left(\frac{\pi}{2} - x\right)$
therefore	$\frac{\mathrm{d}y}{\mathrm{d}x} = \sin x$.

(e) Identify the error made by the student. **[1]**

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Section B
Mechanics

7 A particle P of mass 3 kg is moving on a smooth horizontal surface under the action of two constant horizontal forces $(5\mathbf{i}+3\mathbf{j})$ N and $(a\mathbf{i}+3b\mathbf{j})$ N. The acceleration of P is $(2\mathbf{i}-3\mathbf{j})\,\mathrm{m\,s^{-2}}$.

(a) Find the value of a and the value of b. **[3]**

At time $t = 0$ seconds the velocity of P is $\mathbf{u}\,\mathrm{m\,s^{-1}}$ and at time $t = 5$ seconds the velocity of P is $(7\mathbf{i}-6\mathbf{j})\,\mathrm{m\,s^{-1}}$.

(b) Find, in terms of $\mathbf{i}$ and $\mathbf{j}$, an expression for $\mathbf{u}$. **[2]**

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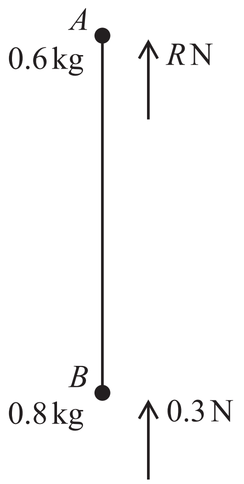

Two particles A and B of masses 0.6 kg and 0.8 kg respectively, are attached to the ends of a light inextensible string. Particle A is held at rest at a fixed point and B hangs vertically below A.

Particle A is now released. As the particles fall the air resistance acting on A has a constant magnitude of R N and the air resistance acting on B has a constant magnitude of 0.3 N. The string remains taut throughout the subsequent motion (see diagram).

The downward acceleration of each of the particles is $9.2\,\mathrm{m\,s^{-2}}$ and the tension in the string is T N.

(a) By applying Newton's second law to B, find the value of T. **[2]**

(b) Hence, or otherwise, find the value of R. **[2]**

9 **In this question you must show detailed reasoning.**

A particle moves in a straight line. Its velocity t seconds after leaving a fixed point on the line is $v\,\mathrm{m\,s^{-1}}$ where $v = 2 + 3t + 0.6t^2$.

Find the distance travelled by the particle from time $t = 0$ until the instant when its acceleration is $15\,\mathrm{m\,s^{-2}}$. **[6]**

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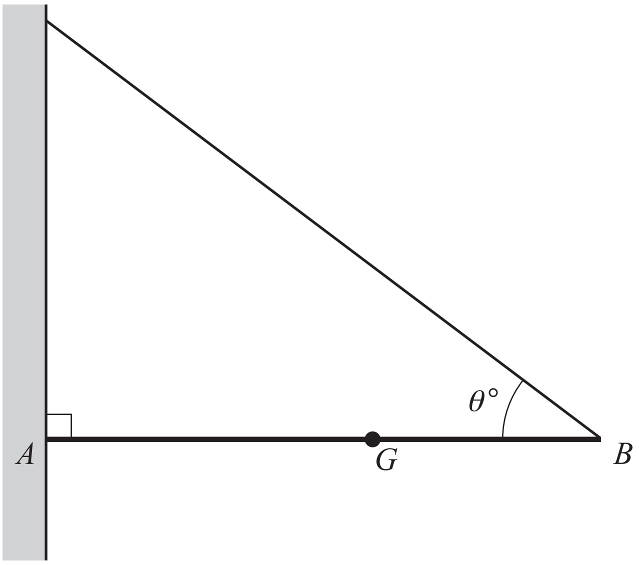

The diagram shows a **non-uniform** rod AB of mass 4 kg and length 2 m. The end A of the rod rests against a rough vertical wall. The rod is held in a horizontal position, perpendicular to the wall, by a light inextensible string attached to the rod at B. The other end of the string is attached to the wall at a point vertically above A. The string is inclined at an angle of $\theta°$ to the horizontal, where $\sin\theta = \frac{4}{5}$.

The rod rests in equilibrium in a vertical plane perpendicular to the wall. The rod's weight acts at the point G on AB.

You are given that the magnitude of the moment of the rod's weight about A is 47.04 N m.

(a) Show that the distance AG is 1.2 m. **[1]**

(b) Find the tension in the string. **[2]**

(c) Determine the magnitude of the contact force exerted on the rod by the wall at A. **[5]**

11 In this question you should take the acceleration due to gravity to be $10\,\mathrm{m\,s^{-2}}$.

The unit vectors **i** and **j** are in a vertical plane with **i** horizontal and **j** vertically upwards.

A small ball P is projected from a point O on horizontal ground into the air. When P is in the air, it is to be modelled as a particle moving under the influence of gravity only.

At time $t = 2$ seconds, P has velocity $(2\mathbf{i} - 8\mathbf{j})\,\mathrm{m\,s^{-1}}$.
The position vector of a point on the trajectory of P is $(x\mathbf{i} + y\mathbf{j})$m relative to O.

(a) Show that $y = 6x - 1.25x^2$. **[5]**

(b) Determine the maximum height of P above the ground during its motion. **[3]**

(c) Determine the following as P passes through the point on its trajectory where $x = 2.5$.

- the speed of P
- the direction of motion of P **[5]**

In reality P does not move only under the influence of gravity but is also subject to air resistance.

(d) Explain how this would affect your answer to part **(b)**. **[1]**

Turn over for question 12

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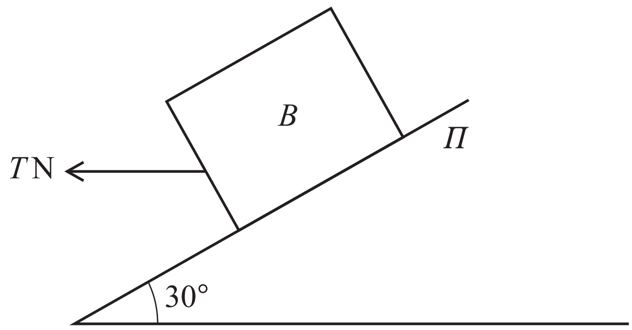
Fig. 1



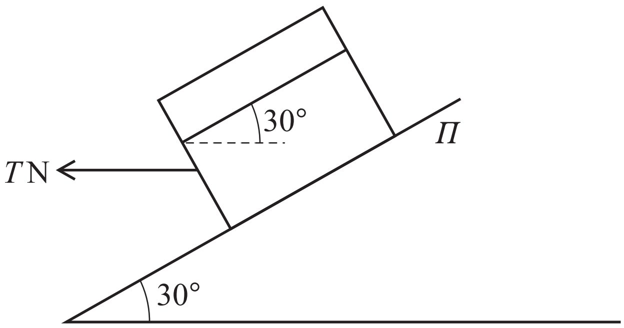
A rectangular block B of mass 10 kg lies at rest in limiting equilibrium on a rough plane Π inclined at 30° to the horizontal. A horizontal force of magnitude T N, acting above a line of greatest slope, is applied to B (see **Fig. 1**).

The coefficient of friction between B and the plane is 0.8.

(a) Show that the value of T is 14.9, correct to **3** significant figures. **[6]**

For the remainder of the question, you should use this value of *T*.

Fig. 2



Block B is now cut at an angle of 30° to the horizontal into two smaller blocks. The upper block has a mass of 4 kg, and the lower block has a mass of 6 kg. The two blocks are held at rest with the lower block on Π. The horizontal force of magnitude T N is now applied to the lower block (see **Fig. 2**).

The two blocks are released from rest and in the subsequent motion the upper block starts to move with acceleration $3.5\,\mathrm{m\,s^{-2}}$.

(b) Determine the coefficient of friction between the two blocks. **[4]**

(c) Show that the lower block does **not** move. **[3]**

END OF QUESTION PAPER