



Oxford Cambridge and RSA

Wednesday 07 October 2020 – Afternoon

AS Level Mathematics B (MEI)

H630/01 Pure Mathematics and Mechanics

Time allowed: 1 hour 30 minutes



You must have:

- the Printed Answer Booklet
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- The acceleration due to gravity is denoted by $g \text{ ms}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **70**.
- The marks for each question are shown in brackets [].
- This document has **8** pages.

ADVICE

- Read each question carefully before you start your answer.

Formulae AS Level Mathematics B (MEI) (H630)

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N}),$$

$$\text{where } {}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Sample variance

$$s^2 = \frac{1}{n-1} S_{xx} \text{ where } S_{xx} = \sum (x_i - \bar{x})^2 = \sum x_i^2 - \frac{(\sum x_i)^2}{n} = \sum x_i^2 - n\bar{x}^2$$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = r) = {}^nC_r p^r q^{n-r}$ where $q = 1 - p$

Mean of X is np

Kinematics

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Answer **all** the questions.

- 1 Celia states that $n^2 + 2n + 10$ is always odd when n is a prime number.

Prove that Celia's statement is false.

[2]

- 2 Fig. 2 shows a quadrilateral ABCD. The lengths AB and BC are 5 cm and 6 cm respectively. The angles ABC, ACD and DAC are 60° , 60° and 75° respectively.

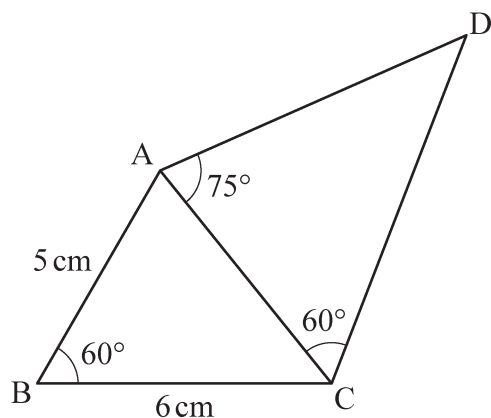


Fig. 2

Calculate the exact value of the length AD.

[4]

- 3 Fig. 3 shows a triangle PQR. The vector \overrightarrow{PQ} is $\mathbf{i} + 7\mathbf{j}$ and the vector \overrightarrow{QR} is $4\mathbf{i} - 12\mathbf{j}$.

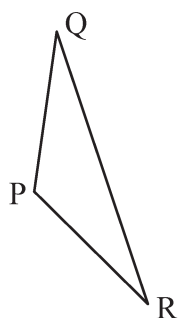


Fig. 3

- (a) Show that the triangle PQR is isosceles.

[3]

The point P has position vector $-3\mathbf{i} - \mathbf{j}$. The point S is added so that PQRS is a parallelogram.

- (b) Find the position vector of S.

[2]

- 4 In this question, the x and y directions are horizontal and vertically upwards respectively.

A particle of mass 1.5 kg is in equilibrium under the action of its weight and forces $\mathbf{F}_1 = \begin{pmatrix} 4 \\ -2 \end{pmatrix}\text{ N}$ and \mathbf{F}_2 .

- (a) Find the force \mathbf{F}_2 . [3]

The force \mathbf{F}_2 is changed to $\begin{pmatrix} 2 \\ 20 \end{pmatrix}\text{ N}$.

- (b) Find the acceleration of the particle. [2]

- 5 Fig. 5.1 shows part of the curve $y = x^{\frac{1}{2}}$. P is the point $(1, 1)$ and Q is the point on the curve with x -coordinate $1 + h$.

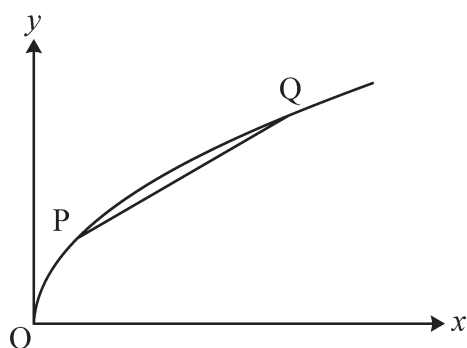


Fig. 5.1

Table 5.2 shows, for different values of h , the coordinates of P, the coordinates of Q, the change in y from P to Q and the gradient of the chord PQ.

x for P	y for P	h	x for Q	y for Q	change in y	gradient PQ
1	1	1				
1	1	0.1	1.1	1.048 809	0.048 809	0.488 088
1	1	0.01	1.01	1.004 988	0.004 988	0.498 756
1	1	0.001	1.001	1.000 500	0.000 500	0.499 875

Table 5.2

- (a) Fill in the missing values for the case $h=1$ in the copy of Table 5.2 in the Printed Answer Booklet. Give your answers correct to 6 decimal places where necessary. [1]
- (b) Explain how the sequence of values in the last column of Table 5.2 relates to the gradient of the curve $y = x^{\frac{1}{2}}$ at the point P. [1]
- (c) Use calculus to find the gradient of the curve at the point P. [2]

6 In this question you must show detailed reasoning.

A particle moves in a straight line. Its velocity $v \text{ ms}^{-1}$ after $t \text{ s}$ is given by $v = t^3 - 5t^2$.

(a) Find the times at which the particle is stationary. [2]

(b) Find the total distance travelled by the particle in the first 6 seconds. [3]

7 In this question you must show detailed reasoning.

A curve has equation $y = 4x^3 - 6x^2 - 9x + 4$.

(a) Sketch the gradient function for this curve, clearly indicating the points where the gradient is zero. [4]

(b) Find the set of values of x for which the gradient function is decreasing. Give your answer using set notation. [2]

8 The point A has coordinates $(-1, -2)$ and the point B has coordinates $(7, 4)$. The perpendicular bisector of AB intersects the line $y + 2x = k$ at P.

Determine the coordinates of P in terms of k . [7]

9 A car travelling in a straight line accelerates uniformly from rest to $V \text{ ms}^{-1}$ in $T \text{ s}$. It then slows down uniformly, coming to rest after a further $2T \text{ s}$.

(a) Sketch a velocity-time graph for the car. [2]

The acceleration in the first stage of the motion is 2.5 ms^{-2} and the total distance travelled is 240 m.

(b) Calculate the values of V and T . [4]

10 An astronaut on the surface of the moon drops a ball from a point 2 m above the surface.

- (a) Without any calculations, explain why a standard model using $g = 9.8\text{ms}^{-2}$ will not be appropriate to model the fall of the ball. [1]

The ball takes 1.6 s to hit the surface.

- (b) Find the acceleration of the ball which best models its motion. Give your answer correct to 2 significant figures. [2]
- (c) Use this value to predict the maximum height of the ball above the point of projection when thrown vertically upwards with an initial velocity of 15ms^{-1} . [2]

11 In this question you must show detailed reasoning.

- (a) A student is asked to solve the inequality $x^{\frac{1}{2}} < 4$.

The student argues that $x^{\frac{1}{2}} < 4 \Leftrightarrow x < 16$, so that the solution is $\{x : x < 16\}$.

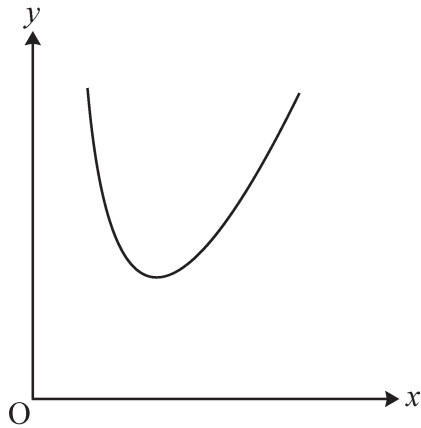
Comment on the validity of the student's argument. [1]

- (b) Solve the inequality $\left(\frac{1}{2}\right)^x < 4$. [3]

- (c) Show that the equation $2\log_2(x+8) - \log_2(x+6) = 3$ has only one root. [5]

12 In this question you must show detailed reasoning.

Fig. 12 shows part of the graph of $y = x^2 + \frac{1}{x^2}$.

**Fig. 12**

The tangent to the curve $y = x^2 + \frac{1}{x^2}$ at the point $(2, \frac{17}{4})$ meets the x -axis at A and meets the y -axis at B. O is the origin.

- (a) Find the exact area of the triangle OAB. [6]
- (b) Use calculus to prove that the complete curve has two minimum points and no maximum point. [6]

END OF QUESTION PAPER

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