PHYSICS ADMISSIONS TEST
Thursday , 5^{th} of November 2020

Time allowed: 2 hours

For candidates applying to Physics, Physics and Philosophy, Engineering, or Materials Science

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Total 26 questions [100 Marks]

Answers should be written on the question sheet in the spaces provided, and you are encouraged to show your working.
You should attempt as many questions as you can.

No tables, or formula sheets may be used.

Answers should be given exactly and in simplest terms unless indicated otherwise.

Indicate multiple-choice answers by circling the best answer.
Partial credit may be given for correct workings in multiple choice questions.

The numbers in the margin indicate the marks expected to be assigned to each question. You are advised to divide your time according to the marks available.

You may take the gravitational field strength on the surface of Earth to be $g \approx 10\,\mathrm{m\,s^{-2}}$

Do NOT turn over until told that you may do so.

1. The stable isotopes of carbon, nitrogen and oxygen are represented symbolically below:

$$^{12}_{6}\mathrm{C}\ ,\ ^{13}_{6}\mathrm{C}\ ,\ ^{14}_{7}\mathrm{N}\ ,\ ^{15}_{7}\mathrm{N}\ ,\ ^{16}_{8}\mathrm{O}\ ,\ ^{17}_{8}\mathrm{O}\ ,\ ^{18}_{8}\mathrm{O}$$

Which of the following statements are true? [2]

1. $^{13}_{6}$C has a larger number of protons than $^{12}_{6}$C. No, both have 6
2. $^{15}_{7}$N has a larger mass than $^{14}_{7}$N. Yes, $15 > 14$
3. $^{16}_{8}$O has a larger nuclear charge than $^{15}_{7}$N. Yes, 8 protons vs 7
4. $^{18}_{8}$O has a larger mass per unit charge than $^{12}_{6}$C. Yes, $18/8 > 12/6$
5. $^{14}_{7}$N has a larger number of neutrons than $^{13}_{6}$C. No, both have 7. $14-7=13-6$

A	B	C	D	E
1, 3, 4	3, 4, 5	2, 3, 4	1, 2, 3	2, 3, 5

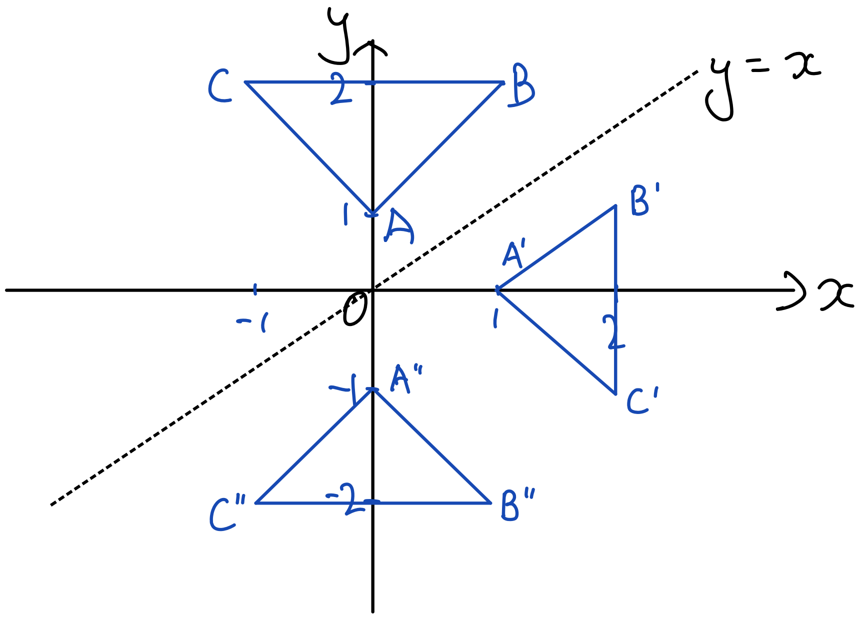
mass no

$^{A}_{Z}X$

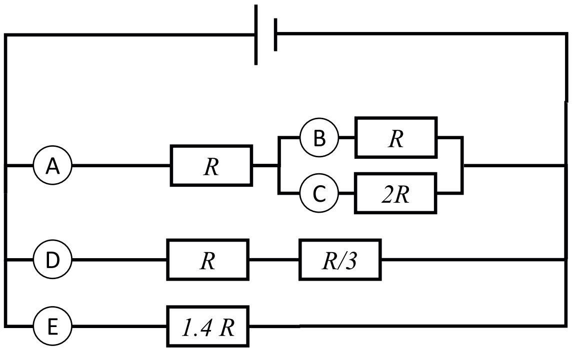
proton no.

2. A triangle ABC has vertices at points in two-dimensional Cartesian co-ordinates $A:(0,1)$, $B:(1,2)$, and $C:(-1,2)$. It is reflected in the line $y=x$ and then rotated around the origin by 90 degrees in a clockwise direction. Which single transformation maps the initial triangle to the final state of the above transformations? [2]

A	B	C	D	E
reflection in $x=0$	reflection in $y=0$	rotation by 180° anti-clockwise around the origin	rotation by 90° anti-clockwise around $(2,0)$	scale factor of -1



3. Which ammeter A, B, C, D, E gives the highest reading? [2]



A	B	C	D	E

Highest reading comes from the lowest resistance

E : $1.4R$

D : $R + \frac{R}{3} = 1.33R$

A : $R + \frac{2R^2}{3R} = \frac{5R}{3} = 1.6R$

B and C each has lower readings than A since $I_A = I_B + I_C$

$\therefore$ Highest current in D

4. Solve $\log_2 x + \log_2(2x+3) = 1$ for x. [2]

A	B	C	D	E
$x = -2$	$x = \frac{1}{2}$	$x = 1$	$x = -2$ and $\frac{1}{2}$	$x = 0$

$$\log_2 x + \log_2(2x+3) = \log_2 2$$

$$\log_2[x(2x+3)] = \log_2 2$$

$$x(2x+3) = 2$$

$$2x^2 + 3x - 2 = 0$$

$$(2x-1)(x+2) = 0$$

$$x = 1/2 \quad \text{or} \quad -2$$

$$x > 0 \quad (\text{logs})$$

$$\therefore \; x = 1/2$$

5. If the gravitational field strength at the Earth's surface is $g_E = 10\,\mathrm{N/kg}$, and at a distance $R > R_E$ from its centre the field strength is $g_R = 2\,\mathrm{N/kg}$, what is the radius of the Earth R_E in terms of R? [2]

A	B	C	D	E
$R/25$	$R/5$	$R/\sqrt{10}$	$R/\sqrt{5}$	$R/\sqrt{2}$

$$g \propto \frac{1}{R^2} \Rightarrow g_E R_E^2 = g_R R^2$$

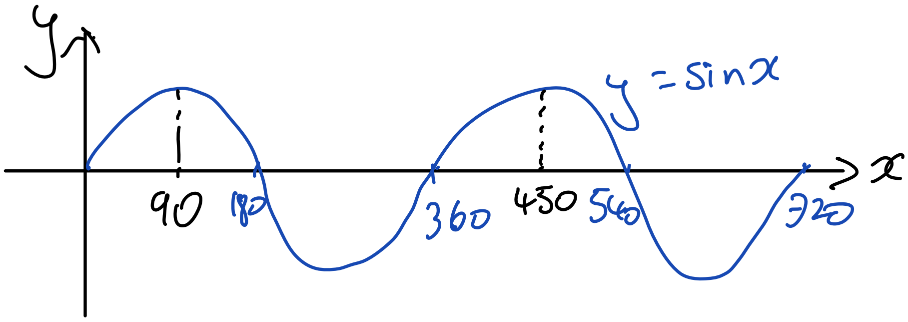
$$R_E = \sqrt{\frac{g_R}{g_E}}\, R$$

$$= \sqrt{\frac{2}{10}}\, R$$

$$= \frac{R}{\sqrt{5}}$$

6. Consider the function $y(x) = \sin(\frac{100}{x})$. The angle is in degrees, so that $\sin(180) = 0$. How many maxima of $y(x)$ occur for $x > 0.1$? [2]

A	**B**	**C**	**D**	**E**
0	1	3	14	∞



$$\frac{100}{x} = 90 + 360n$$

$$x = \frac{100}{90 + 360n} > 0.1$$

$$100 - 9 > 36n$$

$$n < 2.53$$

$$\therefore n = 0, 1, 2$$

7. What is the order, from shortest to longest, of the wavelengths of the peak electromagnetic emission from each of the following objects? [2]

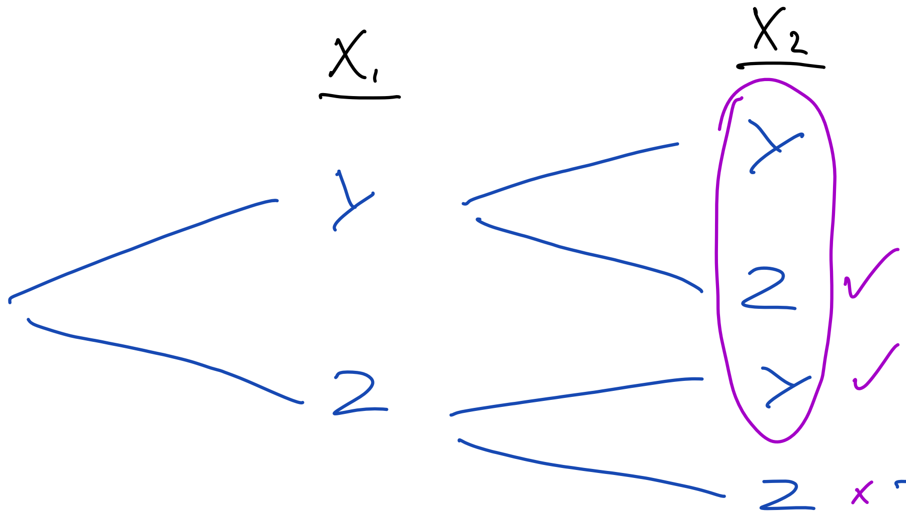
1. an electric torch visible
2. a microwave oven micro
3. a radioactive source gamma
4. a hot cooking stove IR
5. a short-wave radio transmitter. radio

A	**B**	**C**	**D**	**E**
31425	52413	34152	31245	54213

8. A particle of type X decays with equal probability either to a pair of particles of type Y or a pair of particles of type Z. Both Y and Z particles are stable.

The decays of two X particles are observed. A pair of Y particles is found among the decay products. What is the probability that a pair of Z particles is among these decay products? [2]

A	B	C	D	E
1/4	1/3	1/2	2/3	1



2 out of remaining 3 have Z particles

This didn't happen, given that Y is found

9. Ten students need to complete their compulsory practicals for their high school examinations as detailed in the table below:

No. of students	No. of different practicals to complete
2	1
4	2
4	3

The school only has one laboratory in which several different experiments can be set up simultaneously. A maximum of six students are allowed in the school's laboratory for a lesson. Each practical takes one lesson. What is the minimum number of lessons required to complete all the practicals? [2]

A	**B**	**C**	**D**	**E**
3	4	5	6	10

Total no. of practicals: $2(1)+4(2)+4(3)=22$

6 students in 4 lessons = 24 practicals

10. What is the next number in the sequence? 37, 41, 43, 47, 53, 59 [2]

A	**B**	**C**	**D**	**E**
61	62	64	65	67

Prime numbers

11. A stone of average diameter 10 cm is hit with a hammer and splits into pieces. Every time the stone or one of its pieces is hit, it splits into three further pieces of equal volume and similar shape. How many hits will it take before a piece reaches the size of a typical atom? [2]

A	B	C	D	E
9	12	22	56	81

Atom diameter $\approx 10^{-10}$ m

Stone diameter $= 10^{-1}$ m

Stone volume after a hit is reduced to $\frac{V}{3}$

$\Rightarrow$ diameter is reduced to $\frac{10^{-1}}{3^{1/3}}$

After n hits:

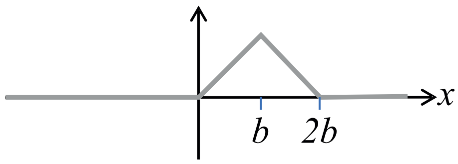
$$\frac{10^{-1}}{(3^{1/3})^n} = 10^{-10}$$

$$10^9 = (3^{1/3})^n$$

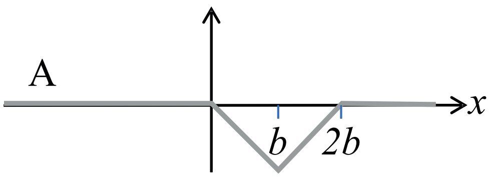
$$n = \frac{\log 10^9}{\frac{1}{3}\log 3}$$

$$= 56.6$$

12. The graph below shows a function $f(x)$.



If a is a constant such that $0 < a < b$, identify the sketch of $g(x) = -f(a - x)$ from the sketches below. [2]

A


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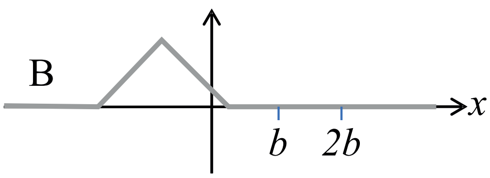
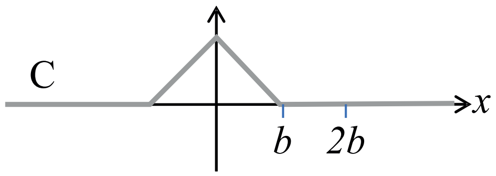
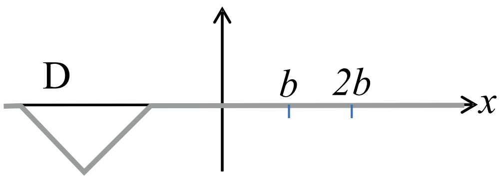
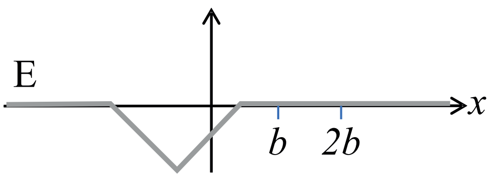
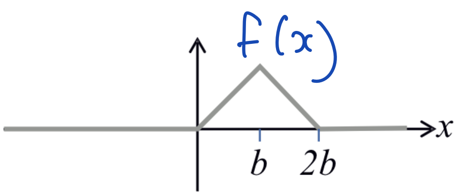


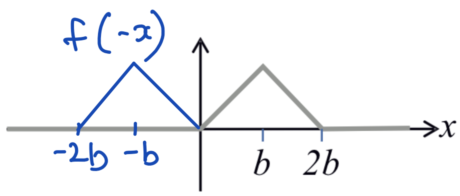
C


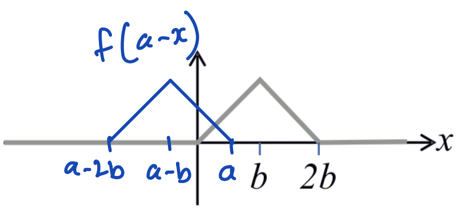
D


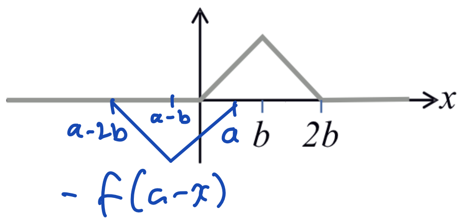
E


A	B	C	D	E









13. Consider a set of masses of three different values m_a, m_b and m_c. Each of the following three combinations have the same total mass.

1. two masses of m_a plus three masses of m_b
2. five masses of m_a plus one mass of m_c
3. two masses of m_a plus one mass of m_b plus one mass of m_c

Find m_b and m_c in terms of m_a. [4]

$$2m_a + 3m_b = 5m_a + m_c = 2m_a + m_b + m_c$$

$$3m_b - m_c = 3m_a \qquad 3m_a = m_b \; *$$

Subs * $3(3m_a) - m_c = 3m_a$

$$m_c = 6m_a$$

14. Find all solutions of the following equation in the range $0 \leq \theta \leq 360°$.

$$4\cos^2(\theta) + 2(\sqrt{3} - 1)\sin(\theta) = 4 - \sqrt{3}$$

[3]

$$4(1-\sin^2\theta) + 2(\sqrt{3}-1)\sin\theta = 4-\sqrt{3}$$

$$4 - 4\sin^2\theta + 2(\sqrt{3}-1)\sin\theta = 4-\sqrt{3}$$

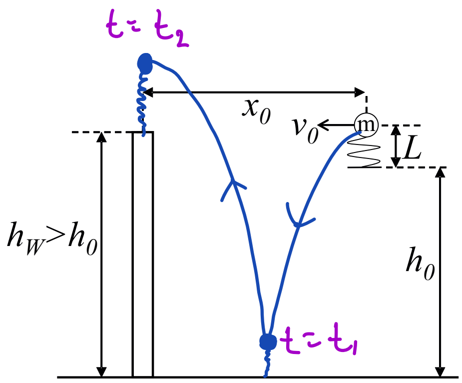
$$4\sin^2\theta - 2(\sqrt{3}-1)\sin\theta - \sqrt{3} = 0$$

$$(2\sin\theta - \sqrt{3})(2\sin\theta + 1) = 0$$

$$\sin\theta = \frac{\sqrt{3}}{2} \quad \text{or} \quad -\frac{1}{2}$$

$$\theta = 60°, 120°, 210°, 330°$$

15.



The above figure depicts a ball of mass m which is equipped on its lower side with a massless spring of uncompressed length L_0 and spring constant k. The spring is initially compressed to a length $L < L_0$, as shown in the figure.

The ball-spring system is either dropped vertically or launched with a horizontal speed v_0 from a height of h_0 at a distance x_0 from a wall of height h_W.

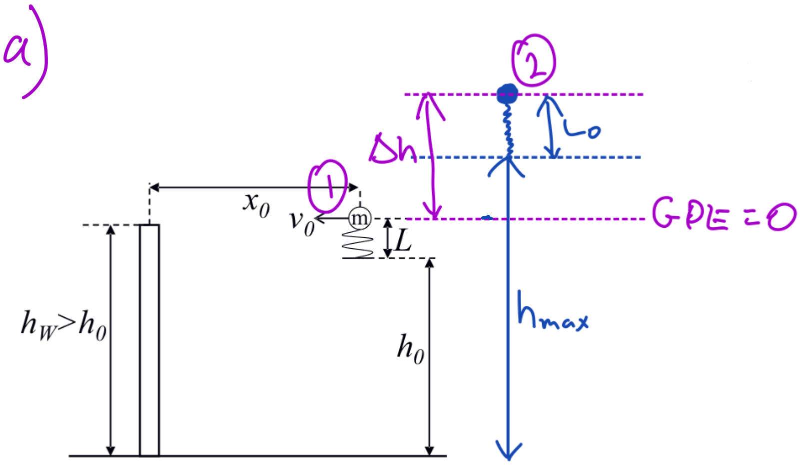
The spring remains compressed until it touches the ground. When it touches the ground the spring is released (by a mechanism not shown on the figure) and expands very quickly back to its uncompressed length L_0 and is then held fixed at length L_0.

The bounce of the ball-spring system on the ground is assumed to be totally elastic.

(a) When the ball is dropped vertically, find the maximum height h_{max} the ball-spring system could reach after its bounce in terms of the spring's compression $\Delta L = L_0 - L$, and the parameters m, g, k and h_0. In analogy to h_0 in the figure, h_{max} should be the vertical distance between the floor and the lower end of the spring after the bounce. You may take $g = 10\,\mathrm{m\,s^{-1}}$. [3]

(b) Find the optimal value for the distance x_0 from which the ball would bounce over a wall with maximum value of h_W. [6]

a)



$EPE_1 = \frac{1}{2}k(\Delta L)^2$

$GPE_1 = 0$

$EPE_2 = 0$

$GPE_2 = mg\Delta h$

$h_0 + L + \Delta h = h_{max} + L_0$

$h_{max} = h_0 - (L_0 - L) + \Delta h$

Conservation of energy: $\frac{1}{2}k(\Delta L)^2 = mg\Delta h$

$\Delta h = \frac{1}{2}\frac{k}{mg}(\Delta L)^2$

$$\Rightarrow h_{max} = h_0 - \Delta L + \frac{k}{2mg}(\Delta L)^2$$

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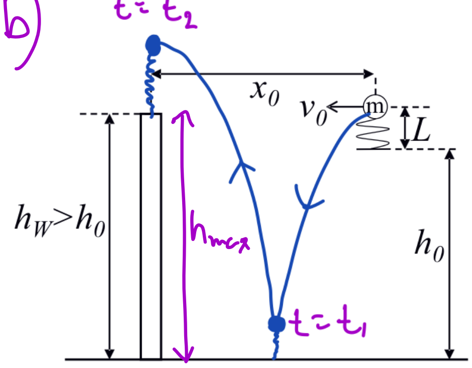
b)



$+\downarrow$ $s = h_0$

$u = 0$

$v = x$

$a = g$

$t = t_1$

$[s = ut + \frac{1}{2}at^2]$

$h_0 = \frac{g}{2}t_1^2$

$t_1 = \sqrt{\frac{2h_0}{g}}$

$+\uparrow$ $s = h_{max} + \Delta L$

$u = x$

$v = 0$

$a = -g$

$t = t_2$

$[s = vt - \frac{1}{2}at^2]$

$h_{max} + \Delta L = \frac{g}{2}t_2^2$

$t_2 = \sqrt{\frac{2}{g}(h_{max} + \Delta L)}$

$= \sqrt{\frac{2}{g}\left(h_0 + \frac{k(\Delta L)^2}{2mg}\right)}$

$\overset{+}{\leftarrow}$ $x_0 = v_0(t_1 + t_2)$

$= v_0\sqrt{\frac{2h_0}{g}} + v_0\sqrt{\frac{2}{g}\left(h_0 + \frac{k(\Delta L)^2}{2mg}\right)}$

$= v_0\sqrt{\frac{2}{g}}\left[\sqrt{h_0} + \sqrt{h_0 + \frac{k(\Delta L)^2}{2mg}}\right]$

16. Given that

$$\frac{\mathrm{d}f(x)}{\mathrm{d}x} = -2x - x^{\frac{1}{2}} + \frac{1}{3}$$
$$\frac{\mathrm{d}g(x)}{\mathrm{d}x} = f(x)$$

determine $g(x)$ such that $f(1) = -1$ and $g(1) = 0$. [4]

$$f(x) = \int \left(-2x - x^{1/2} + \frac{1}{3}\right) dx$$

$$= -x^2 - \frac{2}{3}x^{3/2} + \frac{x}{3} + C$$

$$g(x) = \int \left(-x^2 - \frac{2}{3}x^{3/2} + \frac{x}{3} + C\right) dx$$

$$= \frac{-x^3}{3} - \frac{2}{3} \cdot \frac{2}{5} x^{5/2} + \frac{x^2}{6} + Cx + D$$

$$f(1) = -(1)^2 - \frac{3}{2}(1)^{3/2} + \frac{1}{3} + C = -1$$

$$C = -1 + 1 + \frac{2}{3} - \frac{1}{3}$$

$$= \frac{1}{3}$$

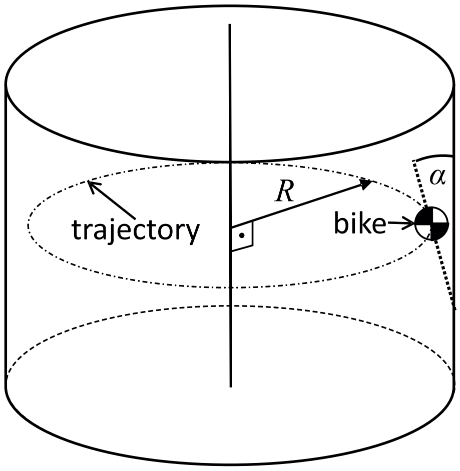
$$g(1) = -\frac{1}{3} - \frac{4}{15} + \frac{1}{6} + \frac{1}{3} + D = 0$$

$$D = \frac{4}{15} - \frac{1}{6}$$

$$= \frac{1}{10}$$

$$\therefore g(x) = -\frac{x^3}{3} - \frac{4}{15}x^{5/2} + \frac{x^2}{6} + \frac{x}{3} + \frac{1}{10}$$

17. As indicated in the figure, a motor bike of total mass m (m is the mass of bike plus rider) is ridden along a horizontal trajectory of radius R on the inside of a cylindrical cage .



The bike and the rider are inclined at an angle α to the wall. The angles α that the bike can make with respect to the walls of the cage are limited by its handle bars to a certain minimum $\alpha_{min} > 0$. The tyres of the bike have a very high co-efficient of friction with the cage so that the tyres can only roll but not slip along the cage.

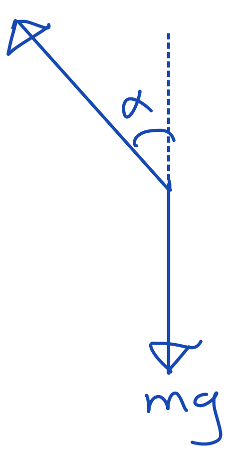
(a) Show in a diagram the forces acting on the bike if it is to maintain a horizontal trajectory as shown in the figure. [2]

(b) At what minimum speed v_{min} must the bike travel if it is not to fall down? [3]

(c) Given that $\alpha_{min} = 30°$, $R = 4\,\text{m}$, $g = 10\ \text{m s}^{-2}$, $m = 250\,\text{kg}$ find a numerical value for v_{min}. [1]

Given very high coefficient of friction so the tyres cannot slip: Assume tyres are stuck to the wall in the vertical direction and ignore friction.

a)



b) $\downarrow\ mg = N\cos\alpha$

$\leftarrow\ N\sin\alpha = \dfrac{mv^2}{R}$

$\dfrac{mg}{\cos\alpha}\cdot\sin\alpha = \dfrac{mv^2}{R}$

$v = \sqrt{Rg\tan\alpha}$

c) $V_{min} = \sqrt{4\times10\times\tan 30}$

$= \sqrt{\dfrac{40}{\sqrt{3}}}$

$= 4.81\,\text{ms}^{-1}$

18. For which x is the following inequality satisfied?

$$\frac{2x^2+3x-2}{2x^2-3x-2} > 0$$

[6]

$$\frac{(2x-1)(x+2)}{(2x+1)(x-2)} > 0$$

multiply by $(2x+1)^2(x-2)^2$

$$(2x-1)(x+2)(2x+1)(x-2) > 0$$

$+ \quad - \quad + \quad - \quad +$

$-2 \quad -\frac{1}{2} \quad \frac{1}{2} \quad 2$

$$\{x < -2\} \cup \left\{-\frac{1}{2} < x < \frac{1}{2}\right\} \cup \{x > 2\}$$

19. The Euler number E_u has no units. It is used in fluid flow calculations. It depends on the pressure P, density ρ, and fluid velocity v such that:

$$E_u = P^a \rho^b v^c$$

where a, b and c are constants.

Find the ratio of $a : b : c$ in its simplest form, where a, b and c are positive or negative integers. [4]

$P : Pa = \frac{N}{m^2} = \frac{kg\,m\,s^{-2}}{m^2} = kg\,m^{-1}s^{-2}$

$\rho : kg\,m^{-3}$

$v : m\,s^{-1}$

$E_u : (kg\,m^{-1}s^{-2})^a (kg\,m^{-3})^b (m\,s^{-1})^c$

$= (kg)^a m^{-a} s^{-2a} (kg)^b m^{-3b} m^c s^{-c}$

$= (kg)^{(a+b)} m^{(-a-3b+c)} s^{(-2a-c)}$

$a + b = 0$

$-a = b$

$-2a - c = 0$

$c = -2a$

$a : b : c$

$a : -a : -2a$

$1 : -1 : -2$

20. Find the coordinates of the point(s) at which the line $y = m(3x - 2)$ is tangent to the curve $y = 9x^2 + 6x - 7$. In the above m is a real constant. [5]

Intersection: $m(3x-2) = 9x^2 + 6x - 7$

$$0 = \underset{a}{9}x^2 + \underset{b}{(6-3m)}x + \underset{c}{(2m-7)}$$

Tangent $\Rightarrow$ one solution

$\Delta = b^2 - 4ac = 0$

$(6-3m)^2 - 4(9)(2m-7) = 0$

$36 - 36m + 9m^2 - 72m + 36(7) = 0$

$9m^2 - 108m + 36(8) = 0$

$m^2 - 12m + 32 = 0$

$(m-4)(m-8) = 0$

$m = 4$ or 8

Same gradient: $3m = 18x + 6$

$$x = \frac{3m-6}{18} = \frac{m-2}{6}$$

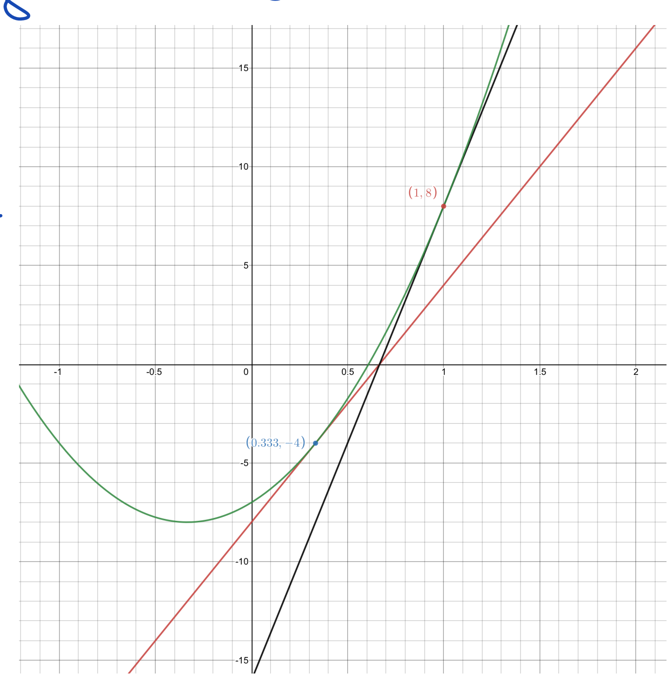
$m=4$: $x = \frac{4-2}{6} = \frac{1}{3}$

$y = 4[3(1/3) - 2] = -4$

$m=8$: $x = \frac{8-2}{6} = 1$

$y = 8[3(1) - 2] = 8$

$\therefore (1, 8)$ and $(1/3, -4)$



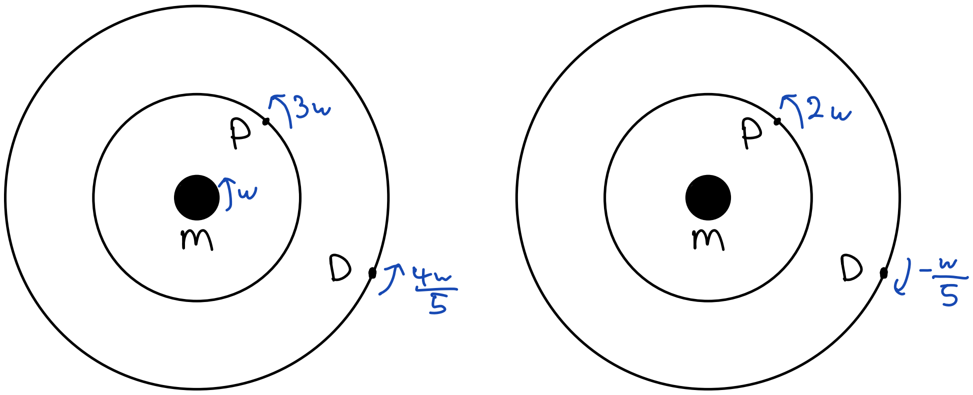
21. Mars' moons Phobos and Deimos are in equatorial, near-circular orbits around Mars. They both orbit in the direction of the planet's rotation. Phobos has an orbital period of 1/3 of a Martian day, and Deimos has an orbital period of 5/4 Martian days. An astronomer on Mars, near the equator, observes both moons during the course of a Martian night.

(a) Would the two moons appear to move in the same direction in the sky? Explain your answer [2]

(b) Describe qualitatively how the phases of the two moons might vary during the night [2]

(c) Would it be possible for the astronomer to see Phobos both rise and set within a single night? [1]

a)



No, relative to the observer, the two moons have opposite angular velocities

b) Phobos goes through all its phases
Deimos moves much slower and there would only be a small change of phase in one night

c) Yes

22. A number can be represented using base N as follows:

$$(x...cba)_N = a \cdot N^0 + b \cdot N^1 + c \cdot N^2 + ... + x \cdot N^w$$

In which base less than 10 is the following equation true?

$$(1101)_N - (313)_N = (344)_N$$

[5]

N^3	N^2	N^1	N^0	
1	1	0	1	$= N^3 + N^2 + 1$
$-$	3	1	3	$-3N^2 - N - 3$
	3	4	4	$3N^2 + 4N + 4$

$$N^3 + N^2 + 1 - 3N^2 - N - 3 = 3N^2 + 4N + 4$$

$$N^3 - 5N^2 - 5N - 6 = 0$$

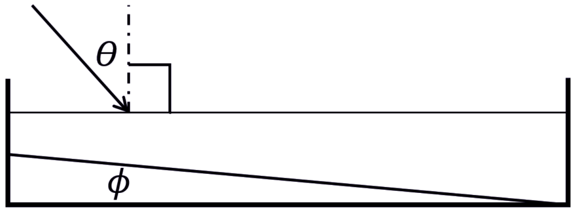
Given $4 < N < 10$

Try $N = 5$: $5^3 - 5(5)^2 - 5(5) - 6 \neq 0$

$N = 6$: $6^3 - 5(6)^2 - 5(6) - 6 = 0$

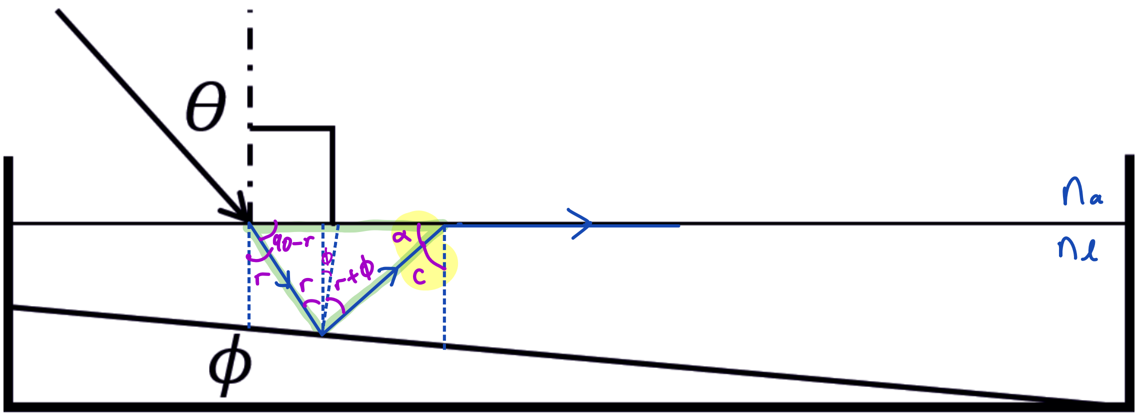
$\therefore N = 6$

23. A monochromatic beam of light travels through air of refractive index n_a and strikes a liquid of refractive index n_l at an angle of incidence θ as shown in the figure below. At the bottom of the tank which contains the liquid is a plane mirror at angle ϕ to the horizontal. The tank can be considered infinitely long.



Beyond a certain value of θ the light no longer leaves the tank after the first reflection in the mirror. Find the value of θ for this case given $\phi = 10°$, $n_a = 1$ and $n_l = \frac{4}{3}$. [5]

You may wish to use the larger diagram below to draw a ray diagram.



$n_l \sin c = n_a \sin 90$

$\sin c = \frac{3}{4}$

$c = 48.6°$

$\alpha = 90 - c$

$\alpha + 90 - r + 2(r+\phi) = 180$

$90 - c + 90 - r + 2r + 2\phi = 180$

$r - c + 2\phi = 0$

$r = 48.6 - 20$

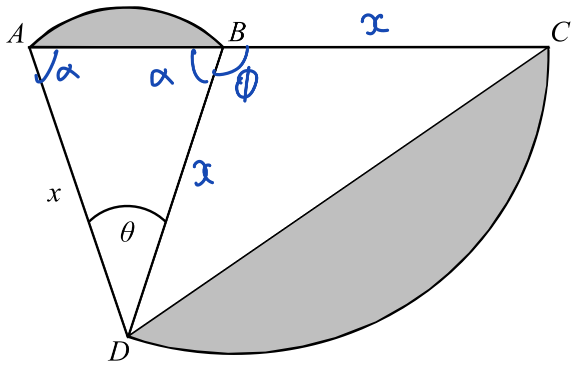
$= 28.6°$

$n_a \sin\theta = n_l \sin r$

$\sin\theta = \frac{4}{3} \sin 28.6$

$\theta = 39.6°$

24. The points A, B and C lie on a straight line. The length of AD is x.



D is the centre of the arc AB, and B is the centre of the arc CD. Find the total shaded area in terms of x and θ. [5]

Small area $= \frac{1}{2}x^2(\theta - \sin\theta)$

large area $= \frac{1}{2}x^2(\phi - \sin\phi)$

$\theta + 2\alpha = \pi$, $\quad \phi + \alpha = \pi$

$\alpha = \pi - \phi$

Subs. α: $\theta + 2\pi - 2\phi = \pi$

$\theta + \pi = 2\phi$

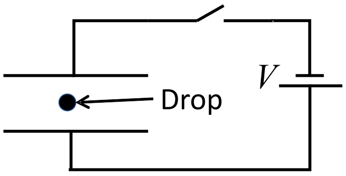
$\phi = \frac{\theta}{2} + \frac{\pi}{2}$

large area $= \frac{1}{2}x^2\left[\frac{\theta}{2} + \frac{\pi}{2} - \sin\left(\frac{\theta}{2} + \frac{\pi}{2}\right)\right]$

$= \frac{1}{2}x^2\left[\frac{\theta}{2} + \frac{\pi}{2} - \cos\left(\frac{\theta}{2}\right)\right]$

Total area $= \frac{1}{2}x^2\left[\frac{3\theta}{2} + \frac{\pi}{2} - \sin\theta - \cos\left(\frac{\theta}{2}\right)\right]$

25. In Millikan's oil drop experiment, shown below, a spherical oil drop with charge $+Q$, radius r and density ρ_{oil} falls between two parallel conducting plates.



(a) Initially the switch is open and the drop is falling with terminal velocity v_t in air of density ρ_{air} and viscosity η. We now measure its terminal velocity.

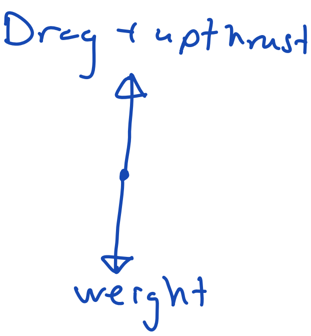
Write down an equation that relates all the forces on the drop while it falls at terminal velocity. [1]

Hint: You may assume that the drag force f_D on the drop is given by $f_D = 6\pi\eta r v_t$ and that the drop also experiences a buoyant force or upthrust f_B equal to the weight of the air displaced by the drop.

(b) The switch is now closed. A uniform electric field E is applied to the drop and it becomes stationary. Write down a new equation relating the forces on the drop. [1]

(c) Derive an equation for Q which does not depend on the radius r. [3]

a)



$$mg = 6\pi\eta r v_t + \rho_a V g$$

$$\rho_o V g - \rho_a V g = 6\pi\eta r v_t$$

$$\frac{4}{3}\pi r^3 g(\rho_o - \rho_a) = 6\pi\eta r v_t$$

$$\frac{2}{3}\pi r^2 g(\rho_o - \rho_a) = 3\pi\eta v_t \quad (1)$$

b) $$\frac{4}{3}\pi r^3(\rho_o - \rho_a) = EQ \quad (2)$$

c) (1): $$r^2 = \frac{3\pi\eta v_t \cdot 3}{2\pi g(\rho_o - \rho_a)}$$

In (2): $$Q = \frac{4\pi r^3}{3E}(\rho - \rho_a) \cdot \left[\frac{9\pi\eta v_t}{2\pi g(\rho_o - \rho_a)}\right]^{3/2}$$

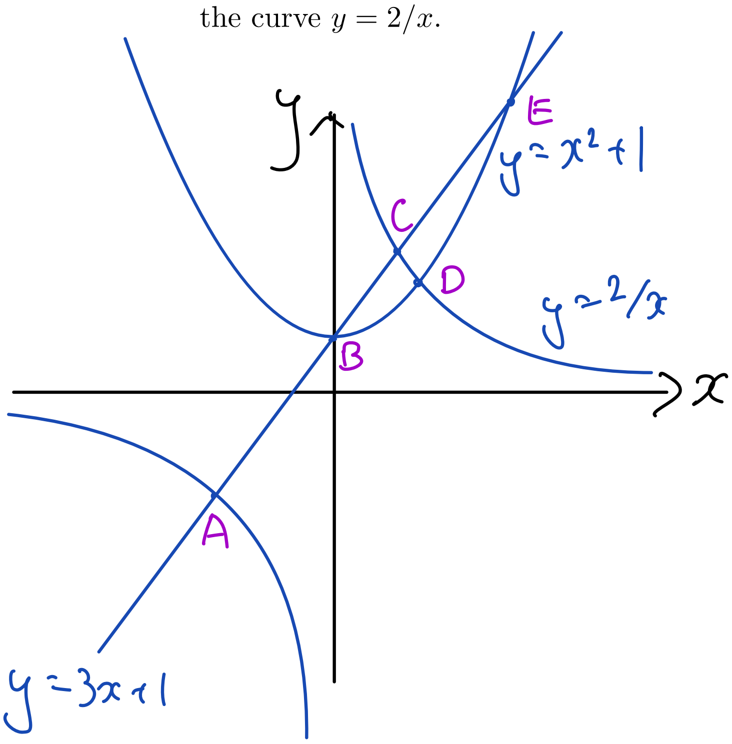
26.

(a) On the same axes, sketch the functions $y = x^2 + 1$, $y = 2/x$, and $y = 3x + 1$. [3]

(b) Determine the exact x coordinates at which any of the graphs intersect and mark these on the x-axis. [3]

(c) Find the exact area enclosed between $y = 3x + 1$ and $y = x^2 + 1$ that is also below the curve $y = 2/x$. [4]

a)



b) A, C: $3x + 1 = \frac{2}{x}$

$3x^2 + x - 2 = 0$

$(3x - 2)(x + 1) = 0$

$\underline{x = 2/3}$, $\underline{x = -1}$

C A

B, E: $3x + 1 = x^2 + 1$

$0 = x^2 - 3x$

$\underline{x = 0}$, $\underline{x = 3}$

B E

D: $x^2 + 1 = \frac{2}{x}$

$x^3 + x - 2 = 0$

$\underline{x = 1}$ D

c) Area $= \int_0^{2/3} (3x + 1)\,dx + \int_{2/3}^1 \frac{2}{x}\,dx - \int_0^1 (x^2 + 1)\,dx$

$= \left[\frac{3x^2}{2} + x\right]_0^{2/3} + 2\left[\ln x\right]_{2/3}^1 - \left[\frac{x^3}{3} + x\right]_0^1$

$= \frac{3}{2} \cdot \frac{4}{9} + \frac{2}{3} + 2\ln 1 - 2\ln\left(\frac{2}{3}\right) - \frac{1}{3} - 1$

$= -2\ln\left(2/3\right)$

$= \underline{2\ln\left(3/2\right)}$