



TMUA/CTMUA

D513/01

PAPER 1

November 2020

*written
solutions*

75 minutes

Additional materials: Answer sheet



INSTRUCTIONS TO CANDIDATES

Please read these instructions carefully, but do not open the question paper until you are told that you may do so.

A separate answer sheet is provided for this paper. Please check you have one. You also require a soft pencil and an eraser.

Please complete the answer sheet with your candidate number, centre number, date of birth, and full name.

This paper is the first of two papers.

There are 20 questions on this paper. For each question, choose the one answer you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

There are no penalties for incorrect responses, only marks for correct answers, so you should attempt **all 20** questions. Each question is worth one mark.

You can use the question paper for rough working or notes, but **no extra paper** is allowed.

You **must** complete the answer sheet within the time limit.

Calculators and dictionaries are NOT permitted.

There is no formulae booklet for this test.

Please wait to be told you may begin before turning this page.

This question paper consists of 21 printed pages and 3 blank pages.

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- 1 Which of the following is an expression for the first derivative with respect to x of

$$\frac{x^3 - 5x^2}{2x\sqrt{x}} = \frac{x^3 - 5x^2}{2x^{3/2}}$$

A $-\frac{\sqrt{x}}{2}$

B $\frac{\sqrt{x}}{4}$

C $\frac{3x - 5}{4\sqrt{x}}$

D $\frac{3\sqrt{x} - 5}{4\sqrt{x}}$

E $\frac{3\sqrt{x} - 10}{3\sqrt{x}}$

F $\frac{3x^2 - 10x}{3\sqrt{x}}$

$$= \frac{1}{2} \left(\frac{x^3}{x^{3/2}} - \frac{5x^2}{x^{3/2}} \right) = \frac{1}{2} (x^{3/2} - 5x^{1/2}) = \frac{1}{2} x^{3/2} - \frac{5}{2} x^{1/2}$$

$$\frac{d}{dx} = \frac{3}{2} \times \frac{1}{2} x^{1/2} - \frac{5}{2} \times \frac{1}{2} x^{-1/2}$$

$$= \frac{3}{4} x^{1/2} - \frac{5}{4} x^{-1/2}$$

$$= \frac{3\sqrt{x}}{4} - \frac{5}{4\sqrt{x}}$$

$$= \frac{3\sqrt{x}\sqrt{x} - 5}{4\sqrt{x}}$$

$$= \frac{3x - 5}{4\sqrt{x}}$$

2 $(2x + 1)$ and $(x - 2)$ are factors of $2x^3 + px^2 + q = f(x)$

What is the value of $2p + q$?

A -10

B $-\frac{38}{5}$

C $-\frac{22}{3}$

D $\frac{22}{3}$

E $\frac{38}{5}$

F 10

By factor theorem, $2x^3 + px^2 + q = 0$ when $x = 2$

and $2x + 1 = 0$

$2x = -1$

$x = -\frac{1}{2}$

So:

$$2(2^3) + p(2^2) + q = 0$$

$$8 \times 2 + 4p + q = 0$$

$$4p + q = -16 \quad (2)$$

$$2 + -\frac{1}{8} + \frac{1}{4}p + q = 0$$

$$-\frac{1}{4} + \frac{1}{4}p + q = 0$$

$$\frac{1}{4}p + q = \frac{1}{4}$$

$$p + 4q = 1$$

$$4p + 16q = 4 \quad (1)$$

(1) - (2) gives $15q = 4 + 16$

$$15q = 20$$

$$q = \frac{20}{15} = \frac{4}{3}$$

so $4q = \frac{16}{3}$

$$p + \frac{16}{3} = 1$$

$$p = -\frac{13}{3}$$

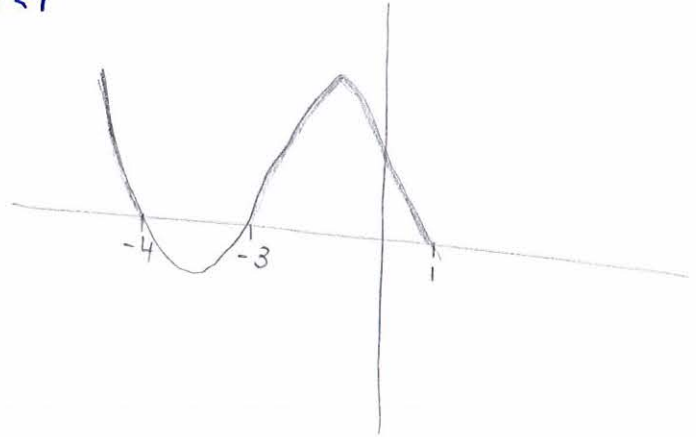
so $2p + q = -\frac{26}{3} + \frac{4}{3} = -\frac{22}{3}$

3 Find the complete set of values of x for which

$$(x+4)(x+3)(1-x) > 0 \quad \text{and} \quad (x+2)(x-2) < 0$$

$x^2 - 4 < 0$ $x^2 < 4$
so $-2 < x < 2$
 $x < 4$ & $-3 < x < 1$

- A $1 < x < 2$
- B** $-2 < x < 1$
- C $-2 < x < 2$
- D $x < -2$ or $x > 1$
- E $x < -4$ or $x > 2$
- F $x < -4$ or $-3 < x < 1$
- G $-4 < x < -2$ or $x > 1$



satisfy both inequalities with $-2 < x < 1$

$$GP \quad a, ar, ar^2, \dots$$

$$AP \quad a, a+d, a+2d, \dots$$

- 4 The 1st, 2nd and 3rd terms of a geometric progression are also the 1st, 4th and 6th terms, respectively, of an arithmetic progression.

The sum to infinity of the geometric progression is 12.

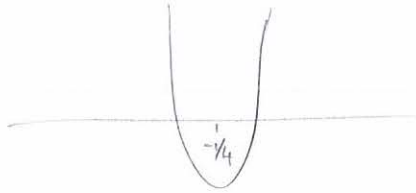
Find the 1st term of the geometric progression.

- | | | | | |
|----------|---|------------|----------------------|-------------------------|
| A | 1 | <u>1st</u> | <u>2nd/4th</u> | <u>3rd/6th</u> |
| B | 2 | $a=a$ | $ar = a+3d$ | $ar^2 = a+5d$ |
| C | 3 | | $r = \frac{a+3d}{a}$ | $r = \frac{a+5d}{a+3d}$ |
| D | 4 | | | |
| E | 5 | | | |
| F | 6 | | | |

$$\begin{aligned} \text{so } (a+3d)^2 &= a(a+5d) \\ a^2 + 6ad + 9d^2 &= a^2 + 5ad \\ ad &= -9d^2 \\ \boxed{a} &= \boxed{-9d} \end{aligned}$$

$$\begin{aligned} \text{so } r &= \frac{-9d + 3d}{-9d} \\ r &= \frac{-6d}{-9d} \\ \boxed{r} &= \boxed{\frac{2}{3}} \end{aligned}$$

$$\begin{aligned} S_{\infty} &= \frac{a}{1-r} \quad \text{so } 12 = \frac{a}{1-\frac{2}{3}} \\ a &= \underline{\underline{4}} \end{aligned}$$



- 5 The curve S has equation

$$y = px^2 + 6x - q$$

where p and q are constants.

S has a line of symmetry at $x = -\frac{1}{4}$ and touches the x -axis at exactly one point.

What is the value of $p + 8q$?

- A 6
 B 18
 C 21
 D 25
 E 38
- $\frac{dy}{dx} = 2px + 6 = 0$ when $x = -1/4$
 $\therefore -\frac{p}{2} + 6 = 0$
 $\frac{p}{2} = 6$
 $p = 12$

only one turning point so discriminant is zero, i.e.

$$36 + 4pq = 0$$

$$36 + 4 \times 12q = 0$$

$$48q = -36$$

$$q = \frac{-36}{48} = \frac{-3}{4}$$

$$\text{So } p + 8q = 12 + 8 \times \frac{-3}{4} = 12 - \frac{24}{4} = 12 - 6 = \underline{\underline{6}}$$

6 Find the maximum value of the function

$$\begin{aligned}f(x) &= \frac{1}{5^{2x} - 4(5^x) + 7} \\&= \frac{1}{(5^x)^2 - 4(5^x) + 7} \\&= \frac{1}{(5^x - 2)^2 + 3}\end{aligned}$$

A $\frac{1}{7}$

B $\frac{1}{4}$

C $\frac{1}{3}$

D 3

E 4

F 7

Maximise the functn by minimising the denominator

ie. when $(5^x - 2)^2 = 0$ so max of $f = \underline{\underline{\frac{1}{3}}}$

7 Given that

and

$$2^{3x} = 8^{(y+3)}$$

$$= (2^3)^{(y+3)}$$

$$= 2^{3y+9}$$

so

$$3x = 3y + 9$$

$$\boxed{x = y + 3}$$

$$4^{(x+1)} = \frac{16^{(y+1)}}{8^{(y+3)}}$$

what is the value of $x + y$?

A) -23

B) -22

C) -15

D) -14

E) -11

F) -10

$$2^{2(x+1)} = \frac{2^{4(y+1)}}{2^{3(y+3)}}$$

$$2^{2x+2} = \frac{2^{4y+4}}{2^{3y+9}}$$

$$2^{2x+2} = 2^{4y+4-3y-9}$$

$$2^{2x+2} = 2^{y-5}$$

$$2x+2 = y-5$$

$$\boxed{2x = y-7}$$

so $y-7 = 2y+6$
 $-13 = y$

then $x = -13+3$
 $= -10$

$$\therefore x+y = -10-13 = \underline{\underline{-23}}$$

8 The function f is defined for all real x as

$$f(x) = (p-x)(x+2) = px + 2p - x^2 - 2x$$

$$= 2p + (p-2)x - x^2$$

Find the complete set of values of p for which the maximum value of $f(x)$ is less than 4.

$$f'(x) = -2x + p - 2$$

A $-2 - 4\sqrt{2} < p < -2 + 4\sqrt{2}$

B $-2 - 2\sqrt{2} < p < -2 + 2\sqrt{2}$

C $-2\sqrt{5} < p < 2\sqrt{5}$

D $-6 < p < 2$

E $-4 < p < 0$

F $-2 < p < 2$

$f(x)$ is a -ve quadratic \therefore stationary point is a maximum

$$f'(x) = 0 \Rightarrow -2x + p - 2 = 0$$

$$-2x = 2 - p$$

$$x = \frac{p-2}{2}$$

so max of f is:

$$f\left(\frac{p-2}{2}\right) = \left(p - \frac{p-2}{2}\right)\left(\frac{p-2}{2} + 2\right) = \left(\frac{2p-p+2}{2}\right)\left(\frac{p-2+4}{2}\right) = \left(\frac{p+2}{2}\right)\left(\frac{p+2}{2}\right)$$

$$= \frac{(p+2)^2}{4}$$

Then $\frac{(p+2)^2}{4} < 4 \Rightarrow -2 < \frac{p+2}{2} < 2 \Rightarrow -4 < p+2 < 4$

$$\Rightarrow \underline{\underline{-6 < p < 2}}$$

- 9 The quadratic expression $x^2 - 14x + 9$ factorises as $(x - \alpha)(x - \beta)$, where α and β are positive real numbers.

Which quadratic expression can be factorised as $(x - \sqrt{\alpha})(x - \sqrt{\beta})$?

- A $x^2 - \sqrt{10}x + 3$
- B $x^2 - \sqrt{14}x + 3$
- C** $x^2 - \sqrt{20}x + 3$
- D $x^2 - 178x + 81$
- E $x^2 - 176x + 81$
- F $x^2 + 196x + 81$

$$\begin{aligned} x^2 - 14x + 9 &= (x - \alpha)(x - \beta) \\ &= x^2 - \beta x - \alpha x + \alpha\beta \\ &= x^2 - (\alpha + \beta)x + \alpha\beta \end{aligned}$$

so $14 = \alpha + \beta$ $\alpha\beta = 9$

$$\sqrt{\alpha\beta} = \sqrt{9} = 3$$

$$\begin{aligned} (x - \sqrt{\alpha})(x - \sqrt{\beta}) &= x^2 - \sqrt{\beta}x - \sqrt{\alpha}x + \sqrt{\alpha}\sqrt{\beta} \\ (\sqrt{\alpha} + \sqrt{\beta})^2 &= \alpha + 2\sqrt{\alpha}\sqrt{\beta} + \beta = \alpha + \beta + 2\sqrt{\alpha\beta} \\ &= 14 + 2 \times 3 \\ &= 14 + 6 = 20 \end{aligned}$$

so expression is $x^2 - (\sqrt{\alpha} + \sqrt{\beta})x + \sqrt{\alpha\beta} = x^2 - \underline{\underline{\sqrt{20}x + 3}}$

10 The following sequence of transformations is applied to the curve $y = 4x^2$

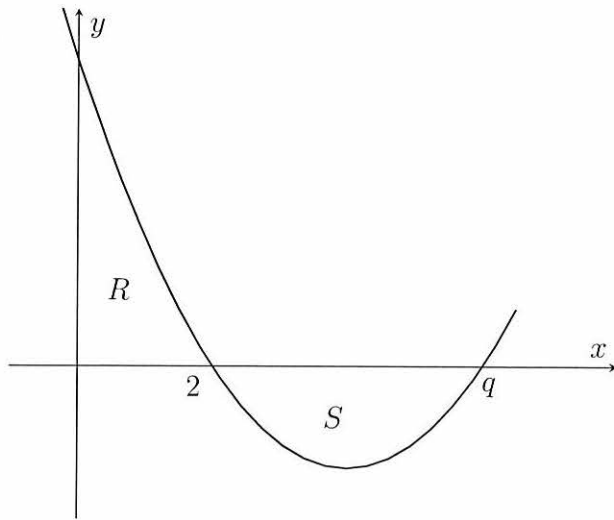
1. Translation by $\begin{pmatrix} 3 \\ -5 \end{pmatrix}$
2. Reflection in the x -axis
3. Stretch parallel to the x -axis with scale factor 2

What is the equation of the resulting curve?

- A $y = -x^2 + 12x - 31$
- B $y = -x^2 + 12x - 41$
- C $y = x^2 + 12x + 31$
- D $y = x^2 + 12x + 41$
- E $y = -16x^2 + 48x - 31$
- F $y = -16x^2 + 48x - 41$
- G $y = 16x^2 - 48x + 31$
- H $y = 16x^2 - 48x + 41$

$$\begin{aligned}
 & \text{reflect} \quad \text{stretch} \quad \text{translate} \\
 & - \left[f \left(\frac{1}{2}x - 3 \right) - 5 \right] = -f \left(\frac{1}{2}x - 3 \right) + 5 \\
 & = -4 \left(\frac{1}{2}x - 3 \right)^2 + 5 \\
 & = -4 \left(\frac{1}{4}x^2 - 3x + 9 \right) + 5 \\
 & = -x^2 + 12x - 36 + 5 \\
 & = \underline{\underline{-x^2 + 12x - 31}}
 \end{aligned}$$

- 11 The quadratic function shown passes through $(2, 0)$ and $(q, 0)$, where $q > 2$.



What is the value of q such that the area of region R equals the area of region S ?

A $\sqrt{6}$
 B 3
 C $\frac{18}{5}$
 D 4
E 6
 F $\frac{33}{5}$

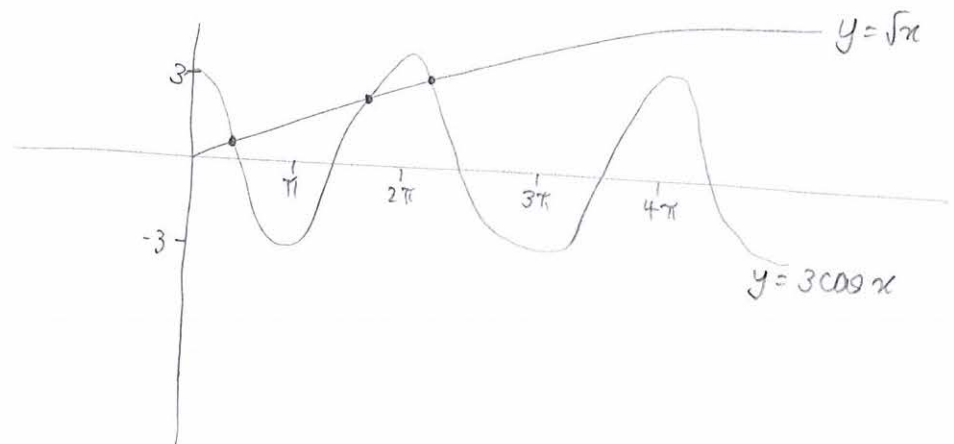
$$\begin{aligned}
 & \int_0^q (x-2)(x-q) dx = 0 \\
 & = \int_0^q x^2 - (q+2)x + 2q dx \\
 & = \frac{x^3}{3} - \frac{(q+2)x^2}{2} + 2qx \Big|_0^q \\
 & = \frac{q^3}{3} - \frac{(q+2)q^2}{2} + 2q^2 \\
 & = \frac{q^3}{3} - \frac{q^3 + 2q^2}{2} + 2q^2 \\
 & = \frac{q^3}{3} - \frac{q^3}{2} - \frac{2q^2}{2} + 2q^2 \\
 & = -\frac{q^3}{6} + q^2 \\
 & = q^2 \left(1 - \frac{q}{6}\right) \\
 & = 0 \quad \text{so } \cancel{q=0} \text{ or } \underline{q=6}
 \end{aligned}$$

12 How many real solutions are there to the equation

$$3 \cos x = \sqrt{x}$$

where x is in radians?

- A 0
- B 1
- C 2
- D 3**
- E 4
- F 5
- G infinitely many



13 Find the coefficient of x^2y^4 in the expansion of $(1+x+y^2)^7$

A 6

B 10

C 21

D 35

E 105

F 210

$$= 1 + \binom{7}{1}(x+y^2) + \binom{7}{2}(x+y^2)^2 + \dots$$

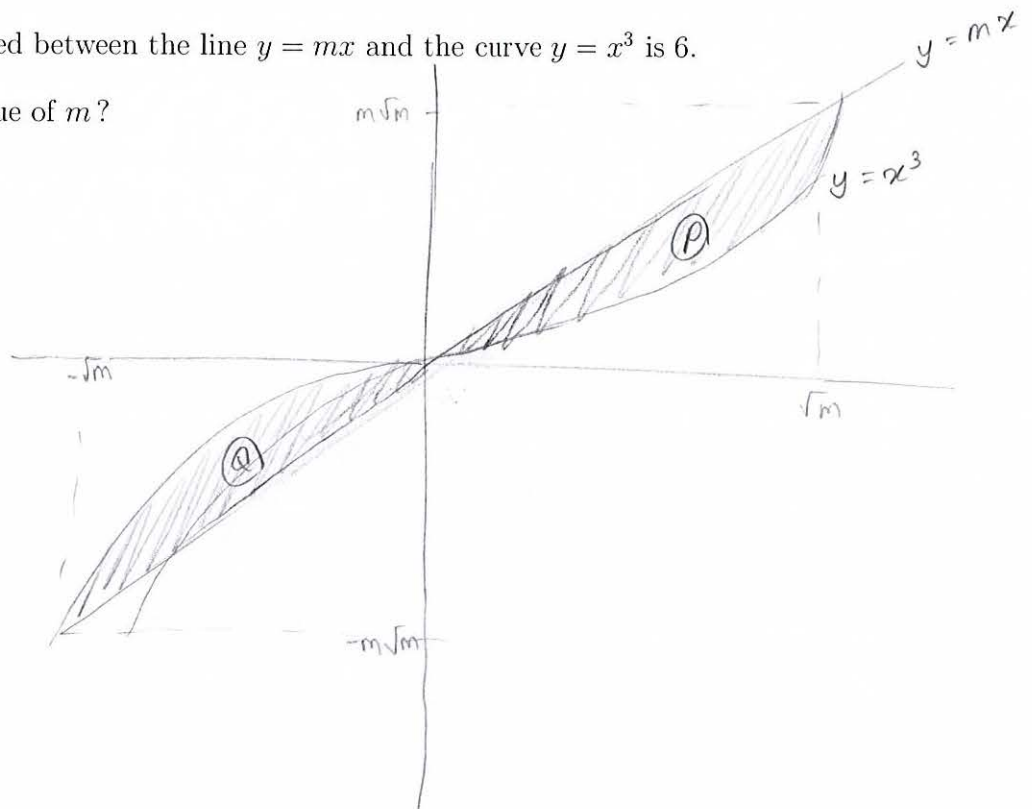
Need the $(x+y^2)^4$ part to get the x^2y^4 term

coefficient $\binom{7}{4} \times \binom{4}{2} = 35 \times 6 = \underline{\underline{210}}$

14 The area enclosed between the line $y = mx$ and the curve $y = x^3$ is 6.

What is the value of m ?

- A 2
- B 4
- C $\sqrt{3}$
- D $\sqrt{6}$
- E $2\sqrt{3}$**
- F $2\sqrt{6}$



lines cross when $x^3 = mx$ i.e. $mx - x^3 = 0$
 $x(m - x^2) = 0$ so $x = 0$ or $\pm\sqrt{m}$
 & $y = (\pm\sqrt{m})^3 = \pm m\sqrt{m}$

P is $\left(\frac{1}{2} \times \sqrt{m} \times m\sqrt{m}\right) - \int_0^{\sqrt{m}} x^3 dx = \frac{1}{2}m^2 - \frac{x^4}{4} \Big|_0^{\sqrt{m}}$
 $= \frac{1}{2}m^2 - \frac{m^2}{4} = \frac{1}{4}m^2$

$P = Q$ so total area = $2 \times \frac{1}{4}m^2 = \frac{1}{2}m^2 = 6$

so $m^2 = 12$

$m = \sqrt{12} = \underline{\underline{2\sqrt{3}}}$

- 15 Find the positive difference between the two real values of x for which

$$(\log_2 x)^4 + 12 \left(\log_2 \left(\frac{1}{x} \right) \right)^2 - 2^6 = 0$$

- A 4
B 16
C $\frac{15}{4}$
D $\frac{17}{4}$
E $\frac{255}{16}$
F $\frac{257}{16}$

$$\log_2 x^4 + 12 (\log_2 x^{-1})^2 - 2^6 = 0$$

$$\log_2 x^4 + 12 (-\log_2 x)^2 - 2^6 = 0$$

let $y = \log_2 x$ then: $y^4 + 12(-y)^2 - 64 = 0$

$$y^4 + 12y^2 - 64 = 0$$

$$(y^2 - 4)(y^2 + 16) = 0$$

so $y^2 = 4$ or ~~$y^2 = -16$~~
 $y = \pm 2$

so $\log_2 x = 2$ or $\log_2 x = -2$
 $x = 4$

$$x = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$$

$$4 - \frac{1}{4} = 3\frac{3}{4} = \underline{\underline{\frac{15}{4}}}$$

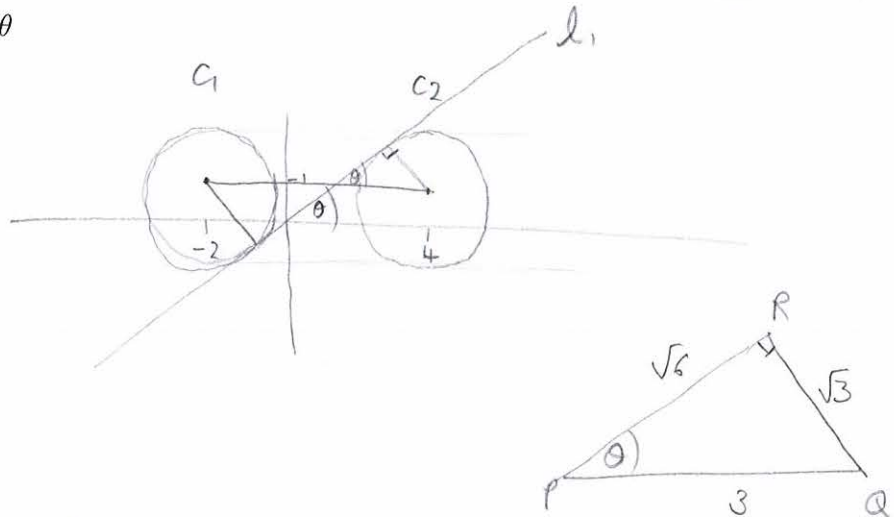
- 16 The circle C_1 has equation $(x + 2)^2 + (y - 1)^2 = 3$ centre $(-2, 1)$ radius $\sqrt{3}$
 The circle C_2 has equation $(x - 4)^2 + (y - 1)^2 = 3$ centre $(4, 1)$ radius $\sqrt{3}$
 The straight line l is a tangent to both C_1 and C_2 and has positive gradient.

The acute angle between l and the x -axis is θ

$$\sqrt{3} \approx 1.7$$

Find the value of $\tan \theta$

- A $\frac{1}{2}$
- B 2
- C $\frac{\sqrt{2}}{2}$**
- D $\sqrt{2}$
- E $\frac{\sqrt{6}}{2}$
- F $\frac{\sqrt{6}}{3}$
- G $\frac{\sqrt{3}}{3}$
- H $\sqrt{3}$

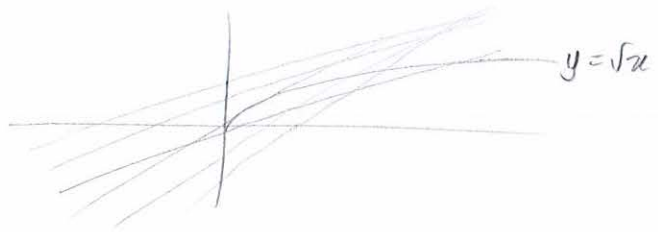


midpoint of centres is $(1, 1)$

$$(PR)^2 = (PQ)^2 - (QR)^2 = 3^2 - 3 = 9 - 3 = 6$$

$$\text{so } PR = \sqrt{6}$$

$$\tan \theta = \frac{\sqrt{3}}{\sqrt{6}} = \frac{\sqrt{2}}{2}$$



- 17 Find the complete set of values of m in terms of c such that the graphs of $y = mx + c$ and $y = \sqrt{x}$ have two points of intersection.

A $0 < m < \frac{1}{4c}$

B $0 < m < 4c^2$

C $m > \frac{1}{4c}$

D $m < \frac{1}{4c}$

E $m > 4c^2$

F $m < 4c^2$

Find when $y = mx + c$ is tangent to $y = \sqrt{x} = x^{1/2}$
i.e. equal gradients

$$\frac{dy}{dx} = \frac{1}{2} x^{-1/2}$$

so $m = \frac{1}{2} x^{-1/2}$

$$\sqrt{x} = \frac{1}{2m}$$

$mx + c = \sqrt{x}$ then $m\left(\frac{1}{4m^2}\right) + c = \frac{1}{2m}$

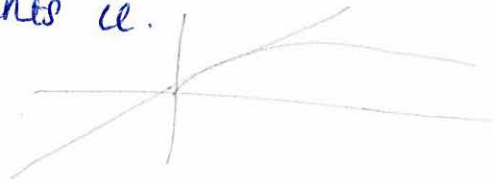
$$\frac{m}{4m^2} + c = \frac{1}{2m}$$

$$\frac{1}{4m} + c = \frac{1}{2m}$$

$$c = \frac{1}{2m} - \frac{1}{4m} = \frac{1}{4m}$$

$$m = \frac{1}{4c}$$

2 solns when m is less steep than the limiting case of equal gradients i.e.



$\therefore \underline{\underline{0 < m < 1/4c}}$

18 Find the number of solutions and the sum of the solutions of the equation

$$1 - 2 \cos^2 x = |\cos x|$$

where $0 \leq x \leq 180^\circ$

- A** Number of solutions = 2 Sum of solutions = 180°
B Number of solutions = 2 Sum of solutions = 240°
C Number of solutions = 3 Sum of solutions = 180°
D Number of solutions = 3 Sum of solutions = 360°
E Number of solutions = 4 Sum of solutions = 240°
F Number of solutions = 4 Sum of solutions = 360°

$$2 \cos^2 x + |\cos x| - 1 = 0$$

When $\cos x$ +ve : $2 \cos^2 x + \cos x - 1 = 0$
 $(2 \cos x - 1)(\cos x + 1) = 0$
So $\cos x = 1/2$ or ~~1~~
i.e. $x = 60^\circ$

For -ve : $|\cos x| = -\cos x$ so $2 \cos^2 x - \cos x - 1 = 0$
 $(2 \cos x + 1)(\cos x - 1) = 0$
 $\cos x = -1/2$ or ~~1~~
i.e. $x = 120^\circ$

\therefore solns are 60° and 120°

$$60^\circ + 120^\circ = \underline{\underline{180^\circ}}$$

19 Find the lowest positive integer for which $x^2 - 52x - 52$ is positive.

A 26 $x^2 - 52x - 52 = 0$ when $x = \frac{52 \pm \sqrt{52^2 + 4 \times 52}}{2}$
B 27
C 51 $= \frac{52 \pm \sqrt{(26 \times 2)^2 + 4 \times 52}}{2}$
D 52
E 53 $= \frac{52 \pm 2\sqrt{26^2 + 52}}{2}$
F 54 $= 26 \pm \sqrt{26^2 + 52}$

$$26^2 + 52 = (26^2 + 2 \times 26 + 1) - 1 = 27^2 - 1 < 27^2$$

so $f(26+27)$ is just 70

20 For how many values of a is the equation

$$(x - a)(x^2 - x + a) = 0$$

satisfied by exactly two distinct values of x ?

A 0

B 1

C 2

D 3

E 4

F more than 4

If 2 distinct values then

either equal roots so discriminant $= 1 - 4a = 0$

$$4a = 1$$

$$a = 1/4$$

or a is also a factor of $x^2 - x + a$

$$\text{i.e. } a^2 - a + a = 0$$

$$a^2 = 0$$

$$a = 0$$

If $a = 0$, then $x = 0, x = 1$

\therefore 2 possible values of a

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