GCE AS MARKING SCHEME

SUMMER 2019

AS (NEW)
FURTHER MATHEMATICS
UNIT 1 FURTHER PURE MATHEMATICS A
2305U10-1

INTRODUCTION

This marking scheme was used by WJEC for the 2019 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

GCE FURTHER MATHEMATICS

AS UNIT 1 FURTHER PURE MATHEMATICS A

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1.	Method 1: $X = A^{-1}B$ det A = 14 $A^{-1} = \frac{1}{14}\begin{pmatrix} 0 & -7 \\ 2 & 3 \end{pmatrix}$ $\therefore X = \frac{1}{14}\begin{pmatrix} 0 & -7 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 5 & 1 \\ 0 & 4 \end{pmatrix}$ $X = \begin{pmatrix} 0 & -2 \\ \frac{5}{7} & 1 \end{pmatrix}$	 M1 B1 M1A1 m1 A1	 M1 valid attempt to find A^{-1} Dep on 1st M1 cao
	Method 2: Let $X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ Then $\begin{pmatrix} 3 & 7 \\ -2 & 0 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 5 & 1 \\ 0 & 4 \end{pmatrix}$ leads to $3a + 7c = 5$ and $-2a + 0c = 0$ $3b + 7d = 1$ and $-2b + 0d = 4$ Solving, $a = 0, b = -2,$ OR $a = 0, c = 5/7$ $c = 5/7, d = 1$ $b = -2, d = 1$	 (M1) (m1) (A1) (m1) (A1) (A1)	 Use of cao cao
2. (a)	$AB: \mathbf{r} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k} + \lambda(4\mathbf{j} + 5\mathbf{k} - 2\mathbf{i} - 3\mathbf{j} + \mathbf{k})$ $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k} + \lambda(-2\mathbf{i} + \mathbf{j} + 6\mathbf{k})$ oe $CD: \mathbf{r} = 7\mathbf{i} - 3\mathbf{k} + \mu(-3\mathbf{i} - \mathbf{j} - 5\mathbf{k} - 7\mathbf{i} + 3\mathbf{k})$ $\mathbf{r} = 7\mathbf{i} - 3\mathbf{k} + \mu(-10\mathbf{i} - \mathbf{j} - 2\mathbf{k})$ oe	M1 A1 A1	Award M1 for either AB or CD If no marks, award SC1 for both AB and CD Condone 1st omission of $\mathbf{r}\ =$ Penalise –1 for 2nd omission of $\mathbf{r}\ =$

(b)	METHOD 1: Directions: $-2\mathbf{i}+\mathbf{j}+6\mathbf{k}$ and $-10\mathbf{i}-\mathbf{j}-2\mathbf{k}$ Checking for perpendicularity, $(-2\mathbf{i}+\mathbf{j}+6\mathbf{k}).(-10\mathbf{i}-\mathbf{j}-2\mathbf{k}) = 20-1-12 = 7$ Because 7≠0, AB and CD are not perpendicular. METHOD 2: Directions: $-2\mathbf{i}+\mathbf{j}+6\mathbf{k}$ and $-10\mathbf{i}-\mathbf{j}-2\mathbf{k}$ Angle between vectors: $\cos\theta = \frac{\text{a.b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}$ $\cos\theta = \frac{7}{\sqrt{41}\sqrt{105}}$ OR $\theta = 83.9^o$ Because $\cos\theta \neq 0$ (OR $\theta \neq 90^o$), AB and CD are not perpendicular.	B1 M1 A1 E1 (B1) (M1) (A1) (E1)	FT (a) For checking scalar product Must refer to '≠0' FT (a) Use Must refer to '≠0' (OR '≠90')
3. a)	$z = 6\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) = 3+3\sqrt{3}i$ $w = 6\left(\cos\frac{-\pi}{6} + i\sin\frac{-\pi}{6}\right) = 3\sqrt{3}-3i$	M1A1 A1	M1 for either z or w or $\frac{11\pi}{6}$
b)	Method 1: $\frac{z}{w} = \frac{3+3\sqrt{3}i}{3\sqrt{3}-3i} = \frac{(3+3\sqrt{3}i)(3\sqrt{3}+3i)}{(3\sqrt{3}-3i)(3\sqrt{3}+3i)}$ $\frac{z}{w} = \frac{9\sqrt{3}+9i+27i-9\sqrt{3}}{27+9}$ $\frac{z}{w} = i$ Method 2: $\left\lvert\frac{z}{w}\right\rvert = \frac{6}{6} = 1$ $\arg\left(\frac{z}{w}\right) = \frac{\pi}{3} - \frac{-\pi}{6} = \frac{\pi}{2}$ $\frac{z}{w} = 1\times\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$ $\frac{z}{w} = i$	M1 A1A1 A1 (B1) (B1) (M1) (A1)	FT (a) A1 num A1 denom Dep on all previous marks awarded FT (a)

4.	When n = 1, $9^1 + 15 = 24$ which is a multiple of 8. Therefore, proposition is true for n = 1. Assume the proposition is true for n = k i.e. $9^k + 15$ is a multiple of 8 or $9^k + 15 = 8N$ Consider n = k+1 $9^{k+1} + 15 = 9(9^k) + 15$ $=9(8N - 15) + 15$ $= 72N - 120$ Each of the two terms are multiples of 8 so therefore is the left hand side. So, if proposition is true for n = k, it's also true for n = k+1. Since we have shown it's true for n = 1, by mathematical induction, it's true for all positive integers n.	B1 M1 M1 A1 A1 A1 A1	 cso

5.	METHOD1: $(x+\frac{1}{2})(x+3)$ $(2x+1)(x+3)$ $2x^2+7x+3$ is the quadratic factor $2x^4-x^3-15x^2+23x+15=0$ $(2x^2+7x+3)(x^2-4x+5)=0$ $\therefore x^2-4x+5=0$ Solving, $x=\frac{4\pm\sqrt{16-20}}{2}$ OR $(x-2)^2=-1$ $x=2+i$ or $x=2-i$ METHOD 2: Use of roots of polynomials to give: $\alpha+\beta-\frac{1}{2}-3=\frac{1}{2} \quad \rightarrow \quad \alpha+\beta=4$ $\alpha\times\beta\times-\frac{1}{2}\times-3=\frac{15}{2} \quad \rightarrow \quad \alpha\beta=5$ $\therefore x^2-4x+5=0$ Solving, $x=\frac{4\pm\sqrt{16-20}}{2}$ OR $(x-2)^2=-1$ $x=2+i$ or $x=2-i$	M1 A1 M1A1 M1 A1 (M1) (A1) (A1) (A1) (M1) (A1)	Allow $x^2+\frac{7}{2}x+\frac{3}{2}$ $\left(x^2+\frac{7}{2}x+\frac{3}{2}\right)$ $(2x^2-8x+10)$ $2x^2-8x+10=0$ cao Award if previous M1 At least 2 equations cao Award if previous M1
6.	$\lvert z-1\rvert=\lvert z-2i\rvert$ $\lvert x+iy-1\rvert=\lvert x+iy-2i\rvert$ $\lvert(x-1)+iy\rvert=\lvert x+i(y-2)\rvert$ $\sqrt{(x-1)^2+y^2}=\sqrt{x^2+(y-2)^2}$ $x^2-2x+1+y^2=x^2+y^2-4y+4$ $-2x+1=-4y+4$ $4y=2x+3$ which is a straight line oe	M1 m1 A1	

7. a)	$\sum_{r=1}^{2m}(r^2+4r+4)$ Use of formulae for $\sum r^2$ and $\sum r$ and $\sum 4$ $= \frac{1}{6}(2m)(2m+1)(4m+1)+4\times\frac{1}{2}(2m)(2m+1)+8m$ $=\frac{1}{6}m(16m^2+12m+2+48m+24+48)$ o.e. $=\frac{1}{3}m(8m^2+30m+37)$	M1 M1 A1 A1	 cao
b)	Substituting appropriate values into their expressions in (a) i.e. $m=5$ and $m=10$ for $2m$ Subtracting values for $r=10$ from $r=20$ $\sum_{r=1}^{2\times 10}(r+2)^2-\sum_{r=1}^{2\times 5}(r+2)^2$ $=\frac{1}{3}\times 10\times(8\times 100+30\times 10+37)-\frac{1}{3}\times 5\times(8\times 25+30\times 5+37)$ $=3145$ *If candidate has used* $\sum_{r=1}^{m}(r+2)^2$ *expression obtained is* $=\frac{1}{6}m(2m^2+15m+37)$ *Then calculations are* Subtracting values for $r=10$ from $r=20$ $\sum_{r=1}^{20}(r+2)^2-\sum_{r=1}^{10}(r+2)^2$ $=\frac{1}{6}\times 20\times(2\times 400+15\times 20+37)$ $-\frac{1}{6}\times 10\times(2\times 100+15\times 10+37)$ $=3145$	M1 m1 A1 A1 (M1) (m1) (A1) (A1)	FT (a) cao FT (a) cao

8.	METHOD 1: Let $\mathbf{r}.\mathbf{n} = 1$ where $\mathbf{n} = p\mathbf{i} + q\mathbf{j} + r\mathbf{k}$	M1	
	$\therefore A: 3p + 5q + 6r = 1$ $B: 5p - q + 7r = 1$ $C: -p + 7q = 1$	A2	A1 for any 1 A2 for all 3
	Substituting $p = 7q - 1$ into A and B gives: $26q + 6r = 4$ $34q + 7r = 6$	m1	Accept working with q or r instead of p
	Solving, $q = \frac{4}{11}$ $r = -\frac{10}{11}$ $p = \frac{17}{11}$	M1 A2	A1 for 2 variables Provided m1 awarded
	Therefore, $\mathbf{r}.\left(\frac{17}{11}\mathbf{i} + \frac{4}{11}\mathbf{j} - \frac{10}{11}\mathbf{k}\right) = 1$ oe	B1	FT p, q, r
	$\frac{17}{11}x + \frac{4}{11}y - \frac{10}{11}z = 1$ oe	B1	FT equation of the plane
	METHOD 2: Let $\mathbf{r}.\mathbf{n} = 1$ where $\mathbf{n} = p\mathbf{i} + q\mathbf{j} + r\mathbf{k}$	(M1)	
	$\mathbf{AB} = 2\mathbf{i} - 6\mathbf{j} + \mathbf{k}$ $\mathbf{BC} = -6\mathbf{i} + 8\mathbf{j} - 7\mathbf{k}$ $\mathbf{CA} = 4\mathbf{i} - 2\mathbf{j} + 6\mathbf{k}$		
	$\mathbf{AB}.\mathbf{n} \rightarrow 2p - 6q + r = 0$ (1) $\mathbf{BC}.\mathbf{n} \rightarrow -6p + 8q - 7r = 0$ (2) $\mathbf{CA}.\mathbf{n} \rightarrow 4p - 2q + 6r = 0$ (3)	(A2)	A1 for any 1 A2 for all 3
	Row operations: (2) + 3(1): $0 - 10q - 4r = 0 \rightarrow r = -\frac{5}{2}q$ (2) + 7(1): $8p - 34q + 0 = 0 \rightarrow p = \frac{17}{4}q$	(m1)	
	Let $q = 4$, $\therefore p = 17$, $q = 4$, $r = -10$ oe	(A2)	A1 for 2 variables
	$\mathbf{a}.\mathbf{n} = (3 \times 17) + (5 \times 4) + (6 \times -10) = 11$	(B1)	
	Therefore, $p = \frac{17}{11}$ $q = \frac{4}{11}$ $r = -\frac{10}{11}$		
	$\mathbf{r}.\left(\frac{17}{11}\mathbf{i} + \frac{4}{11}\mathbf{j} - \frac{10}{11}\mathbf{k}\right) = 1$ oe	(B1)	FT p, q, r
	$\frac{17}{11}x + \frac{4}{11}y - \frac{10}{11}z = 1$ oe	(B1)	FT equation of the plane

9. a)	$u + iv = (x + iy)^2 - 1$ $u + iv = x^2 - y^2 + 2ixy - 1$ Comparing coefficients Imaginary parts: $v = 2xy$ (given) Real parts: $u = x^2 - y^2 - 1$	M1 A1 m1 A1	 Both correct
b)	Putting $y = 3x$ $v = 2x \times 3x = 6x^2$ $u = x^2 - 9x^2 - 1$ $(= -8x^2 - 1)$ Eliminating x, the equation of the locus Q is $u = -8\left(\frac{v}{6}\right) - 1$ oe simplified	M1 A1 M1 A1	FT (a) A1 for both u and v cao
10. a)	From equation 1: $\alpha\beta = \frac{r}{p}$ $\alpha + \beta = -\frac{q}{p}$ Sum of roots = $2\alpha + 2\beta$ $(= 2(\alpha + \beta))$ Sum of pairs = $\alpha\beta + \alpha(\alpha + \beta) + \beta(\alpha + \beta)$ $(= \alpha\beta + (\alpha + \beta)^2)$ Triples = $\alpha\beta(\alpha + \beta)$ $\therefore \frac{-b}{a} = 2 \times \left(-\frac{q}{p}\right) = -\frac{2q}{p}$ $\frac{c}{a} = \frac{r}{p} + \left(-\frac{q}{p}\right)^2 = \frac{r}{p} + \frac{q^2}{p^2}$ $\frac{-d}{a} = \frac{r}{p} \times \left(-\frac{q}{p}\right) = -\frac{qr}{p^2}$ The equation is $x^3 + \frac{2qx^2}{p} + \left(\frac{r}{p} + \frac{q^2}{p^2}\right)x + \frac{qr}{p^2} = 0$ oe	 B1 B2 B2 B1	 Both correct B1 for 2 correct B1 for 2 correct FT 2nd B2 above
b)	From equation 2: $2\alpha\gamma = -\frac{r}{p}$ $\alpha = \frac{r}{p\beta} = -\frac{r}{2p\gamma}$ $\therefore \frac{1}{\beta} = -\frac{1}{2\gamma}$ $\therefore \beta = -2\gamma$	 B1 M1 A1	 Convincing