GCE AS MARKING SCHEME

SUMMER 2019

AS (NEW)
FURTHER MATHEMATICS
UNIT 3 FURTHER MECHANICS A
2305U30-1

INTRODUCTION

This marking scheme was used by WJEC for the 2019 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

GCE FURTHER MATHEMATICS

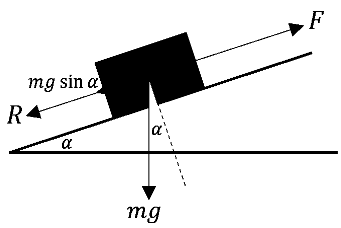
AS UNIT 3 FURTHER MECHANICS A

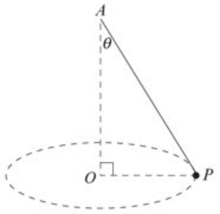
SUMMER 2019 MARK SCHEME

Q1	Solution	Mark	Notes
(a)	Use of Hooke's Law	M1	
	$21 = \frac{\lambda x}{0{\cdot}15}$	A1	
	$\lambda = 35 \quad (\text{N})$	A1	cao
		[3]	
(b)	Using expression for EE **or** KE	M1	
	Energy at start, $\text{EE} = \frac{\lambda x^2}{2(0{\cdot}15)} \quad \left(\frac{35(0{\cdot}09)^2}{2(0{\cdot}15)} = 0 \cdot 945\right)$	A1	FT λ and x from (a)
	Energy at end, $\text{KE} = \frac{1}{2}(0 \cdot 1)v^2 \quad (= 0 \cdot 05v^2)$	A1	
	Conservation of energy	M1	Used with EE **and** KE
	$0 \cdot 05v^2 = 0 \cdot 945$		
	$v = 4 \cdot 3$ (ms^{-1}) $\quad (2 \text{ sig. figs})$	A1	cao
		[5]	
	Total for Question 1	**8**	

Q2	Solution	Mark	Notes
(a)	$\mathbf{a} = \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}$ $\mathbf{a} = 6t\mathbf{i} - 8\mathbf{j} - 2e^{-t}\mathbf{k}$	M1 A1 **[2]**	Correct differentiation of at least one term All correct
(b)	$\mathbf{F} = m\mathbf{a} = 0\cdot5(6t\mathbf{i} - 8\mathbf{j} - 2e^{-t}\mathbf{k})$ $\mathbf{F}.\mathbf{v} = (3t \times 3t^2) + (-4 \times -8t) + (-e^{-t} \times 2e^{-t})$ $\mathbf{F}.\mathbf{v} = 9t^3 + 32t - 2e^{-2t}$	B1 M1 A1 **[3]**	FT **a** from part (a) Correct method for dot product cao
(c)	$\mathbf{v}.\mathbf{v} = (3t^2)^2 + (-8t)^2 + (2e^{-t})^2$ $\mathrm{KE} = \frac{1}{2}m\,\mathbf{v}.\mathbf{v}$ $\mathrm{KE} = \frac{1}{2\times2}\,(9t^4 + 64t^2 + 4e^{-2t})$ $\left(\mathrm{KE} = \frac{9}{4}t^4 + 16t^2 + e^{-2t}\right)$	M1 m1 A1 **[3]**	 cao
(d)	rate of work (power) $= \frac{\mathrm{d}}{\mathrm{d}t}(\mathrm{KE})$ $\frac{\mathrm{d}}{\mathrm{d}t}(\mathrm{KE}) = \frac{\mathrm{d}}{\mathrm{d}t}\frac{1}{4}(9t^4 + 64t^2 + 4e^{-2t})$ $= 9t^3 + 32t - 2e^{-2t} = \mathbf{F}.\mathbf{v}$	B1 B1 **[2]**	Any equivalent statement, mathematical or otherwise Convincing
	Total for Question 2	**10**	

Q3	Solution	Mark	Notes
(a)	Comparison of coefficients $\mathbf{i}$ $60 + 168t = 62 + 160t$ $t = 0 \cdot 25$ $\mathbf{j}$ $2 + 132t = pt$ $t = 0 \cdot 25 \Rightarrow p = 140$ $\mathbf{k}$ $4 = 3 + qt$ $t = 0 \cdot 25 \Rightarrow q = 4$	M1 A1 A1 A1 **[4]**	Comparison attempted for any component Convincing Convincing
(b)	$\mathbf{r}_B - \mathbf{r}_A = (2 - 8t)\mathbf{i} + (-2 + 8t)\mathbf{j} + (-1 + 4t)\mathbf{k}$ $AB^2 = (2 - 8t)^2 + (-2 + 8t)^2 + (-1 + 4t)^2$ $(AB^2 = 144t^2 - 72t + 9)$	M1 A1 **[2]**	 Correct method. Must lose **i, j, k** and be linear expressions
(c)	$AB^2 = 144t^2 - 72t + 9 = 0 \cdot 6^2$ $AB^2 = 144t^2 - 72t + 8 \cdot 64 = 0$ $(50t^2 - 25t + 3 = 0)$ Solving quadratic $t = 0 \cdot 2, (0 \cdot 3)(\text{hours})$ Alarms first activated at 9.12 (a.m.) <u>Alternative Solution</u> Taking out a common factor of $(4t - 1)^2$ from the unsimplified form in (b) $AB^2 = (4t - 1)^2[(-2)^2 + 2^2 + 1] = 9(4t - 1)^2$ $9(4t - 1)^2 = 0 \cdot 6^2$ or $3(4t - 1) = 0 \cdot 6$ Solving quadratic $t = 0 \cdot 2, (0 \cdot 3)$ (hours) Alarms first activated at 9.12 (a.m.)	M1 m1 A1 A1 **[4]** (M1) (m1) (A1) (A1) **([4])**	FT quadratic from (b) Attempt to solve resulting in at least one value of t. FT quadratic from (b) provided it is of the form $a(4t - 1)^2$ Attempt to solve resulting in at least one value of t.
	Total for Question 3	**10**	

Q4	Solution	Mark	Notes
(a)	 		
	At maximum speed $F = R$ (N2L with $a = 0$)	M1	Used
	$F = \frac{P}{v}$	M1	Used, si
	$2000 = \frac{80 \times 1000}{v}$		
	$v = 40 \quad (\mathrm{ms}^{-1})$	A1	cao
		[3]	
(b)	 		
	$F = \frac{0 \cdot 8 \times 80 \times 1000}{20} \quad (= 3200)$	B1	si
	N2L	M1	All forces, dim. correct F and R opposing
	$F - R - mg\sin\alpha = ma$	A1	Allow one error
	$F - 2000 - 1200g \times \frac{1}{20} = 1200a$	A1	FT candidates F
	$a = 0 \cdot 51 \quad (\mathrm{ms}^{-2})$	A1	cao
		[5]	
(c)	Any valid reason eg. Resistance could vary with speed.	E1 **[1]**	
	Total for Question 4	**9**	

Q5	Solution	Mark	Notes
(a)	 Resolve vertically $490\sqrt{3}\cos\theta = 75g$ $\cos\theta = \frac{\sqrt{3}}{2}$ $\theta = 30^0$	 M1 A1 A1 **[3]**	 Convincing
(b)	N2L towards centre $490\sqrt{3}\sin\theta = 75a$ $490\sqrt{3}\sin\theta = 75(1.4)^2 r$ length of chain $= l$ $l\sin\theta = r$ $490\sqrt{3}\sin\theta = 75(1.4)^2 l\sin\theta$ $l = \frac{490\sqrt{3}}{75(1.4)^2}$ $l = 5 \cdot 77(3502\ldots)$ (m)	M1 A1 m1 m1 A1 **[5]**	 $a = \omega^2 r$ cao Accept $\frac{10\sqrt{3}}{3}$
	Total for Question 5	**8**	

Q6	Solution	Mark	Notes
(a)	Conservation of energy	M1	KE **and** PE in dim. correct equation
	$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mgr(1-\cos\theta)$	A1 A1	KE PE
	$v^2 = u^2 - 2gr(1-\cos\theta)$		
	$v^2 = 60g - 20g(1-\cos\theta)$	A1	
	or $v^2 = \begin{cases} 40g + 20g\cos\theta \\ 20g(2+\cos\theta) \end{cases}$	**[4]**	
(b)	N2L towards centre	M1	Dim. correct equation, R and $mg\cos\theta$ opposing
	$R - mg\cos\theta = \frac{mv^2}{10}$	A1	
	$R = \frac{m}{10}(40g + 20g\cos\theta) + mg\cos\theta$	m1	Substitute their v^2
	$R = 4mg + 2mg\cos\theta + mg\cos\theta$ $R = 4mg + 3mg\cos\theta$		
	$R = mg(4 + 3\cos\theta)$	A1	Convincing
		[4]	
(c)	Test for $R = 0$	M1	si
	$mg(4 + 3\cos\theta) = 0$ $\cos\theta = -\frac{4}{3}$, which is not possible (i.e. car will perform loop)	A1	Convincing
	<u>Alternative solution</u>	**[2]**	
	Consider R when $\theta = 180$ $R = mg\big(4 + 3(-1)\big) = mg > 0$	(M1) (A1)	si Convincing
	(i.e. car will perform loop)	**([2])**	
(d)	Loss in PE $= mg(30 - 28)$ $= 2mg$	B1	
	Work-energy principle $\frac{mg}{32} \times d = 2mg$ $d = 2 \times 32$	M1	Used, $F \times d = E$
	$d = 64$ (m)	A1	cao
		[3]	
	Total for Question 6	**13**	

Q7	Solution	Mark	Notes
(a)	Conservation of momentum	M1	Allow 1 sign error
	$mu + 0 = mv_A + mv_B$	A1	All correct
	Restitution	M1	Allow one sign error
	$v_B - v_A = -e(-u)$	A1	All correct, any form
	$v_A + v_B = u$ $-v_A + v_B = eu$		
	$2v_A = (1 - e)u$	m1	One variable eliminated
	$v_A = \frac{1}{2}(1 - e)u$	A1	cao, oe
	$v_B = \frac{1}{2}(1 + e)u$	A1	cao, oe
		[7]	
(b)	Loss in KE $= \frac{1}{2}mu^2 - \frac{1}{2}m\left[\left(\frac{1}{4}u\right)^2 + \left(\frac{3}{4}u\right)^2\right]$	M1	
	$= \frac{1}{2}mu^2\left(1 - \frac{5}{8}\right) = \frac{3}{16}mu^2 \quad (\text{J})$	A1	cao
		[2]	

(c)	Velocity of B after 2nd collision $= \frac{1}{2}(1 - e_1) \times \frac{3}{4}u$	M1	FT (a)
	For no further collisions to occur,		
	Vel. of B after 2nd collision $\geq$ Vel. of A after 1st collision		
	$\frac{1}{2}(1 - e_1) \times \frac{3}{4}u \geq \frac{1}{4}u$	M1	FT (a)
	$3 - 3e_1 \geq 2$		
	$e_1 \leq \frac{1}{3}$	A1	Convincing
		[3]	
	<u>Alternative solution</u> Vel. of B after 2nd collision $= \frac{1}{2}(1 - e_1) \times \frac{3}{4}u$	(M1)	FT (a)
	If $e_1 \leq \frac{1}{3}$ then $1 - e_1 \geq \frac{2}{3}$		
	Vel. of B after 2nd collis $\geq \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}u = \frac{1}{4}u = v_A$	(M1)	FT (a)
	Vel. of B after 2nd collision $\geq$ Vel. of A after 1st collision	(M1)	Convincing
	Total for Question 7	**12**	