GCE A LEVEL – NEW

1305U40-1

S19-1305U40-1

MONDAY, 3 JUNE 2019 – MORNING

FURTHER MATHEMATICS – A2 unit 4
FURTHER PURE MATHEMATICS B

2 hours 30 minutes

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ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.
Answer **all** questions.
Sufficient working must be shown to demonstrate the **mathematical** method employed.
Answers without working may not gain full credit.
Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

INFORMATION FOR CANDIDATES

The maximum mark for this paper is 120.
The number of marks is given in brackets at the end of each question or part-question.
You are reminded of the necessity for good English and orderly presentation in your answers.

Reminder: *Sufficient working must be shown to demonstrate the* ***mathematical*** *method employed.*

1. A complex number is defined by $z = 3 + 4\mathrm{i}$.

(a) Express z in the form $z = r\mathrm{e}^{\mathrm{i}\theta}$, where $-\pi \leqslant \theta \leqslant \pi$. [3]

(b) (i) Find the Cartesian coordinates of the vertices of the triangle formed by the cube roots of z when plotted in an Argand diagram. Give your answers correct to two decimal places.

(ii) Write down the geometrical name of the triangle. [5]

2. *(a)* Show that $3\sin x + 4\cos x - 2$ can be written as $\dfrac{6t+2-6t^2}{1+t^2}$, where $t = \tan\left(\dfrac{x}{2}\right)$. [2]

(b) Hence, find the general solution of the equation $3\sin x + 4\cos x - 2 = 3$. [7]

3. *(a)* Determine whether or not the following set of equations

$$\begin{pmatrix} 2 & -7 & 2 \\ 0 & 3 & -2 \\ -7 & 8 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

has a unique solution, where a, b, c are constants. [3]

(b) Solve the set of equations

$$\begin{aligned} x + 8y - 6z &= 5, \\ 2x + 4y + 6z &= -3, \\ -5x - 4y + 9z &= -7. \end{aligned}$$

Show all your working. [5]

4. *(a)* Given that $y = \cot^{-1}x$, show that $\dfrac{dy}{dx} = \dfrac{-1}{x^2+1}$. [5]

(b) Express $\dfrac{6x^2-10x-9}{(2x+3)(x^2+1)}$ in terms of partial fractions. [5]

(c) Hence find $\displaystyle\int \frac{6x^2-8x-6}{(2x+3)(x^2+1)}dx$. [5]

(d) Explain why $\displaystyle\int_{-2}^{5} \frac{6x^2-8x-6}{(2x+3)(x^2+1)}dx$ cannot be evaluated. [1]

5. *(a)* Show that $\sin\theta - \sin3\theta$ can be expressed in the form $a\cos b\theta\sin\theta$, where a, b are integers whose values are to be determined. [3]

(b) Find the mean value of $y = 2\cos2\theta\sin\theta + 7$ between $\theta = 1$ and $\theta = 3$, giving your answer correct to two decimal places. [5]

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6. Solve the differential equation

$$\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 10y = 0,$$

where $\dfrac{dy}{dx} = 1$ and $\dfrac{d^2y}{dx^2} = 8$ when $x = 0$. [10]

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7. *(a)* Write down the Maclaurin series expansion for $\ln(1-x)$ as far as the term in x^3. [2]

(b) Show that $-2\ln\left(\dfrac{1-x}{(1+x)^2}\right)$ can be expressed in the form $ax + bx^2 + cx^3 + \ldots$, where

a, b, c are integers whose values are to be determined. [4]

8. The curve *C* has polar equation

$$r = \sin 2\theta, \quad \text{where} \quad 0 < \theta \leqslant \frac{\pi}{2}.$$

(a) Find the polar coordinates of the point on *C* at which the tangent is parallel to the initial line. Give your answers correct to three decimal places. [9]

(b) Write the coordinates of this point in Cartesian form. [1]

9. *(a)* Given that $y = \sin^{-1}(\cos\theta)$, where $0 \leqslant \theta \leqslant \pi$, show that $\dfrac{\mathrm{d}y}{\mathrm{d}\theta} = k$, where the value of k is to be determined. [4]

(b) Find the value of the gradient of the curve $y = x^3 \tan^{-1} 4x$ when $x = \dfrac{\pi}{2}$. [4]

(c) Find the equation of the normal to the curve $y = \tanh^{-1}(1-x)$ when $x = 1{\cdot}7$. [6]

10. Given the differential equation

$$\sec x \frac{\mathrm{d}y}{\mathrm{d}x} + y \operatorname{cosec} x = 2$$

and $x = \frac{\pi}{2}$ when $y = 3$, find the value of y when $x = \frac{\pi}{4}$. [8]

11. *(a)* Find the area of the region enclosed by the curve $y = x\sinh x$, the x-axis and the lines $x = 0$ and $x = 1$. [4]

(b) The region *R* is bounded by the curve $y = \cosh 2x$, the x-axis and the lines $x = 0$ and $x = 1$. Find the volume of the solid generated when *R* is rotated through four right-angles about the x-axis. [4]

(c) **Using your answer to part *(b)***, find the total volume of the solid generated by rotating the region bounded by the curve $y = \cosh 2x$ and the lines $x = -1$ and $x = 1$. [1]

12. *(a)* Evaluate $\int_3^4 \frac{1}{\sqrt{x^2 - 4}}\,\mathrm{d}x$, giving your answer correct to three decimal places. [3]

(b) Given that $\int_1^2 \frac{k}{9 - x^2}\,\mathrm{d}x = \ln\frac{25}{4}$, find the value of k. [5]

(c) Show that $\int \frac{(\cosh x - \sinh x)^3}{\cosh^2 x + \sinh^2 x - \sinh 2x}\,\mathrm{d}x$ can be expressed as $-\mathrm{e}^{-x} + c$, where c is a constant. [6]

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