GCE A LEVEL – NEW

1305U50-1

S19-1305U50-1

THURSDAY, 6 JUNE 2019 – AFTERNOON

FURTHER MATHEMATICS – A2 unit 5
FURTHER STATISTICS B

1 hour 45 minutes

1305U501
01

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator;
- statistical tables (RND/WJEC Publications).

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.
Answer **all** questions.
Sufficient working must be shown to demonstrate the **mathematical** method employed.
Answers without working may not gain full credit.
Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

INFORMATION FOR CANDIDATES

The maximum mark for this paper is 80.
The number of marks is given in brackets at the end of each question or part-question.
You are reminded of the necessity for good English and orderly presentation in your answers.

Reminder: *Sufficient working must be shown to demonstrate the* ***mathematical*** *method employed.*

1. A coffee shop produces biscuits to sell. The masses, in grams, of the biscuits follow a normal distribution with mean μ. Eight biscuits are chosen at random and their masses, in grams, are recorded. The results are given below.

32·1	29·9	31·0	31·1	32·5	30·8	30·7	31·5

(a) Calculate a 95% confidence interval for μ based on this sample. [7]

(b) Explain the relevance or otherwise of the Central Limit Theorem in your calculations. [1]

2. The continuous random variable X is uniformly distributed over the interval $(\theta - 1, \theta + 5)$, where θ is an unknown constant.

(a) Find the mean and the variance of X. [2]

(b) Let $\bar{X}$ denote the mean of a random sample of 9 observations of X. Find, in terms of $\bar{X}$, an unbiased estimator for θ and determine its standard error. [4]

3. The rules for the weight of a cricket ball state:

"The ball, when new, shall weigh not less than **155·9 g**, nor more than **163 g**."

A company produces cricket balls whose weights are normally distributed. It wants 99% of the balls it produces to be an acceptable weight.

(a) What is the largest acceptable standard deviation? [3]

The weights of the cricket balls are in fact normally distributed with mean 159·5 grams and standard deviation 1·2 grams. The company also produces tennis balls. The weights of the tennis balls are normally distributed with mean 58·5 grams and standard deviation 1·3 grams.

(b) Find the probability that the weight of a randomly chosen cricket ball is more than three times the weight of a randomly chosen tennis ball. [6]

4. Rugby players sometimes use protein powder to aid muscle increase. The monthly weight gains of rugby players taking protein powder may be modelled by a normal distribution having a standard deviation of 40 g and a mean which may depend on the type of protein powder they consume. A rugby team coach gives the same amount of protein powder over a trial month to 22 randomly selected players.

Protein powder *A* was used by 12 players, randomly selected, and their mean weight gain was 900 g. Protein powder *B* was used by the other 10 players and their mean weight gain was 870 g.

Let μ_A and μ_B be the mean monthly weight gains, in grams, of the populations of rugby players who use protein powder *A* and protein powder *B* respectively.

(a) Calculate a 98% confidence interval for $\mu_A - \mu_B$. [4]

(b) In the given context, what can you conclude from your answer to part *(a)*? Give a reason for your answer. [2]

(c) Find the confidence level of the largest confidence interval that would lead the coach to favour protein powder *A* over protein powder *B*. [4]

(d) State one non-statistical assumption you have made in order to reach these conclusions. [1]

1305U501 03

5. To qualify as a music examiner, a trainee must listen to a series of performances by 8 randomly chosen students. An experienced examiner and the trainee both award scores for each of the 8 performances. In order for the trainee to qualify, there must not be a significant difference between the average scores given by the experienced examiner and the trainee.

(a) Explain why the Wilcoxon signed rank test is appropriate. [2]

The scores awarded are shown below.

Student	A	B	C	D	E	F	G	H
Experienced Examiner	108	109	92	95	145	148	134	120
Trainee	114	116	95	93	137	144	133	110

(b) (i) Carry out an appropriate Wilcoxon signed rank test on this dataset, using a 5% significance level.

(ii) What conclusion should be reached about the suitability of the trainee to qualify? [9]

TURN OVER

6. A manufacturer of batteries for electric cars claims that an hour of charge can power a certain model of car to travel for an average of 123 miles. An electric car company and a consumer, Hopcyn, both wish to test the validity of the manufacturer's claim.

(a) Explain why Hopcyn may want to use a one-sided test and why the car company may want to use a two-sided test. [2]

To test the validity of this claim, Hopcyn collects data from a random sample of 90 drivers of this model of car to see how far they travelled, X miles, on an hour of charge. He produced the following summary statistics.

$$\sum x = 11007 \qquad\qquad \sum x^2 = 1361913$$

(b) (i) Assuming Hopcyn uses a one-sided test, state the hypotheses.

(ii) Test at the 5% significance level whether the manufacturer's claim is correct. [8]

7. Nathan believes that shearers from Wales can shear more sheep, on average, in a given time than shearers from New Zealand. He takes a random sample of 8 shearers from Wales and 7 shearers from New Zealand. The numbers below indicate how many sheep were sheared in 45 minutes by the 15 shearers.

Wales:	60	53	42	38	37	36	31	28
New Zealand:	39	35	27	26	17	16	15	

Use a Mann-Whitney U test at the 1% significance level to test whether Nathan is correct. You must state your hypotheses clearly and state the critical region. [7]

8. The random variable X has probability density function

$$f(x) = 1 + \frac{3\lambda x}{2} \qquad \text{for} \quad -\frac{1}{2} \leqslant x \leqslant \frac{1}{2},$$

$$f(x) = 0 \qquad \text{otherwise,}$$

where λ is an unknown parameter such that $-1 \leqslant \lambda \leqslant 1$.

(a) (i) Find $\mathrm{E}(X)$ in terms of λ.

(ii) Show that $\mathrm{Var}(X) = \dfrac{16 - 3\lambda^2}{192}$. [6]

(b) Show that $\mathrm{P}(X > 0) = \dfrac{8 + 3\lambda}{16}$. [2]

In order to estimate λ, n independent observations of X are made. The number of positive observations obtained is denoted by Y and the sample mean is denoted by $\bar{X}$.

(c) (i) Identify the distribution of Y.

(ii) Show that T_1 is an unbiased estimator for λ, where

$$T_1 = \frac{16Y}{3n} - \frac{8}{3}.$$ [4]

(d) (i) Show that $\mathrm{Var}(T_1) = \dfrac{64 - 9\lambda^2}{9n}$.

(ii) Given that T_2 is also an unbiased estimator for λ, where

$$T_2 = 8\bar{X},$$

find an expression for $\mathrm{Var}(T_2)$ in terms of λ and n.

(iii) Hence, giving a reason, determine which is the better estimator, T_1 or T_2. [6]

END OF PAPER

BLANK PAGE

BLANK PAGE