GCE AS MARKING SCHEME

SUMMER 2022

AS (NEW)
FURTHER MATHEMATICS
UNIT 2 FURTHER STATISTICS A
2305U20-1

INTRODUCTION

This marking scheme was used by WJEC for the 2022 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

WJEC GCE AS FURTHER MATHEMATICS

UNIT 2 FURTHER STATISTICS A

SUMMER 2022 MARK SCHEME

Qu. No.	Solution	Mark	Notes
1 (a)	$p = 0.0099$	B1	
(b)	$E(X) = (0 \times 0.9+)2 \times 0.09 + 100 \times 0.0099 + 1000 \times 0.0001$	M1	FT "their p" Allow one slip
	$E(X) = 1.27$	A1	
	$Var(X) = (0^2 \times 0.9+)2^2 \times 0.09 + 100^2 \times 0.0099 + 1000^2 \times 0.0001 - 1.27^2$	M1	FT "their p" and "their$E(X)$" Allow one slip
	$Var(X) = 197.7(471)$	A1	Accept 198 from correct working
(c)(i)	£1.28	B1	FT their E(X)
(ii)	Valid explanation. e.g. People may be willing to pay for the excitement of the lottery. The lottery may be raising money for charity. People don't often make decisions based on mathematics. People could win a lot of money.	E1	
		Total [7]	

2 (a)	$S_{xy} = 113.16 - \frac{62.8 \times 19.4}{10}$ $S_{xy} = -8.672$ $S_{xx} = 413.44 - \frac{62.8^2}{10}$ $S_{xx} = 19.056$ $S_{yy} = 46.16 - \frac{19.4^2}{10}$ $S_{yy} = 8.524$ $r = \frac{-8.672}{\sqrt{19.056 \times 8.524}}$ $r = -0.68(0427 \ldots)$	B1 B1 B1 B1	B1 for each of S_{xy},S_{xx} and S_{yy}. B1 for r.
(b)	$H_0: \rho = 0$ $H_1: \rho \neq 0$ 5% two tail critical value = –0.6319 Since –0.6804 < –0.6319 reject H_0. It suggests that the rate of unemployment and the rate of wage inflation are not independent.	B1 B1 B1 E1	FT their r Accept in context Or CV = 0.6319 Or 0.6804 > 0.6319 Only award E1 if previous three B1 awarded E0 for categorical statements
(c)	Valid comment. e.g. This should cast doubt on Amy's opinion based on her answer in (b) Valid suggestion. e.g. She could look at more countries. She could come to different conclusions for different countries. She could consider more regions within each country	E1 E1	FT their conclusion from (b)
(d)	The underlying distribution is bivariate normal. The data come from a bivariate normal distribution.	E1	
		Total [11]	

3 (a)	Total number of baskets, T, is $Po\big((2.1+1.9)\times 4\big)$ or $Po(16)$ or $Po(2.1\times 4+1.9\times 4)$ $P(T=20)=\dfrac{16^{20}\times e^{-16}}{20!}$ $=0.0559$	M2 m1 A1	M1 for Poisson and adding. M1 for multiplying by 4. Dependent on M2 Use of formula or calculator cao
(b) (i)	Exponential distribution Mean time between baskets= standard deviation $=$ $\frac{1}{2.1}\times 12$ 5.7 minutes.	B1 M1 A1	 Must be clear that 5.7… is mean AND standard deviation
(b) (ii)	P (Klay doesn't score for the rest of the quarter) = $e^{(-1.9\times 0.75)}$ = 0.2405 Alternative solution $\lambda=1.425$ P(X = 0) = 0.2405	M1 A1 (M1) (A1)	 M1 for Po(1.9×0.75) SC1 for $(e^{(-2.1\times 0.75)}=)0.207$
(c)	Let F be the number of free throws he misses. F ~$B(530, 0.04)$ $P(F>25)=1-P(F\leq 25)$ $=0.169(1214\ldots.)$	M1 A1 **Total [11]**	

4 (a)	The pdf must be positive (or zero) $f(r) \geq 0$	B1	B1 for implying that the pdf must be positive or zero (or cannot be negative)
	Therefore $(b-4) \geq 0$ $b \geq 4$	B1	B1 for Correct statement leading to correct conclusion. ALTERNATIVE B1 for "If $b < 4$, $f(r)$is negative." B1 for stating that is not possible.
4 (b) (i)	$\int_1^4 kr(4-r)dr = 1$ $\int_1^4 (4kr - kr^2)dr = 1$	M1	M1 Attempt at integration at least one power of r increasing by 1. Limits and = 1 not required here.
	$k\left[\frac{4r^2}{2} - \frac{r^3}{3}\right]_1^4 = 1$	A1	A1 Correct integration.
	$k\left[\left(\frac{64}{2} - \frac{64}{3}\right) - \left(\frac{4}{2} - \frac{1}{3}\right)\right] = 1$	m1	m1 substitution of correct limits and =1.
	$k = \frac{1}{9}$ *ag	A1	Convincing

4 (b) (ii)	$F(r) = \frac{1}{9}\int_1^r t(4-t)dt$	M1	M1 Attempt at integrating $f(t)$ at least one power of t increasing by 1. Limits not required here.
	$= \frac{1}{9}\left[\frac{4t^2}{2} - \frac{t^3}{3}\right]_1^r$	A1	A1 Correct integration.
	$= \frac{1}{9}\left[2r^2 - \frac{r^3}{3} - \left(2 - \frac{1}{3}\right)\right]$	m1	m1 substituting correct limits Condone upper limit = x for m1 only
	$= \frac{1}{9}\left(2r^2 - \frac{r^3}{3} - \frac{5}{3}\right)$		
	$= \frac{1}{27}(6r^2 - r^3 - 5)$	A1	oe Mark final expression for $1 \leq r \leq 4$
(iii)	$P(2 \leq R \leq 3) = F(3) - F(2)$	M1	oe
	$= \frac{22}{27} - \frac{11}{27}$		
	$= \frac{11}{27}$	A1	FT their $F(r)$ for equivalent difficulty and provided probability is valid.
		Total [12]	

5	Let the random variable X be the number of 6s thrown from 3 dice. If the dice are unbiased then $X \sim B(3, \frac{1}{6})$	B1	si (implied by at least 3 correct expected frequencies)
	H_0: The data can be modelled by the Binomial distribution $B(3, \frac{1}{6})$. H_1: The data cannot be modelled by the Binomial distribution $B(3, \frac{1}{6})$.	B1	or equivalent
	<table><tr><td>Number of sixes</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>Observed</td><td>625</td><td>384</td><td>81</td><td>10</td></tr><tr><td>Expected</td><td>636.574</td><td>381.944</td><td>76.389</td><td>5.093</td></tr></table>	M1 A1	At least one correct. All correct.
	Use of χ^2 stat = $\sum \frac{(O-E)^2}{E}$ or $\sum \frac{O^2}{E} - N$	M1	Must see at least 2 terms added
	$= \frac{(625-636.574)^2}{636.574} + \frac{(384-381.944)^2}{381.944} + \frac{(81-76.389)^2}{76.389} + \frac{(10-5.093)^2}{5.093}$	m1	$\frac{625^2}{636.574} + \frac{384^2}{381.944} + \frac{81^2}{76.389} + \frac{10^2}{5.093} - 1100$
	$= 5.23$	A1	Accept anything which rounds to 5.2
	DF = 3 5% CV = 7.815	B1 B1	Accept other test levels. 1% CV = 11.345 10% CV = 6.251
	Since $5.23 < 7.815$ we cannot reject H_0. There is insufficient evidence at the 5% level to conclude that the set of dice are not fair.	B1 E1	FT their χ^2 Only award E1 if all five previous B1 awarded E0 for categorical statements
		Total [11]	

6 (a)	H_0: Social media usage is independent of age. H_1: Social media usage is not independent of age	B1	
(b)	$\frac{1266\times352}{1953}$ $=228.18$ *ag	B1	oe
(c)	$s=\frac{(412-342.27)^2}{342.27}$ $s=14.2(0595699...)$	M1 A1	
(d)	$(4-1)\times(2-1)=3$ degrees of freedom. 5% CV = 7.815 Add χ^2 contributions $29.34+14.21+0.06+62.94+54.07+26.18+0.11+115.99=302.90$ Since $302.91>7.815$ we can reject H_0. There is (strong) evidence to suggest that social media usage is not independent of age.	B1 B1 M1 A1 B1 E1	 M1A1 if statement along the lines of "one contribution is > 7.815" FT provided $\chi^2>7.815$ Only award E1 if previous three B1 awarded and part (a) correct
(e)	Valid explanation. e.g. The *p* value would not lead to rejecting H_0, which is the incorrect conclusion.	E1 **Total [11]**	

7 (a)	$b = \frac{96.60984}{88.42142}$	M1	
	$b = 1.09(26\ldots)$	A1	Accept 1.1
	$a = \frac{2738.656}{30} - 1.09(26\ldots) \times \frac{2850.836}{30}$	M1	FT their 'b' for M1
	$a = -12.5(39\ldots)$	A1	FT their 'b', following A0. Answer correct to 3sf
	$y = -12.5 + 1.09x$	A1	A1 FT 'their' gradient and intercept provided at least one M1 awarded.
(b)	Africa because 70 is out of the data set for Asia, The data points for Africa are closer to a straight line than those for the Arab World.	E1 E1	
		Total [7]	