GCE AS MARKING SCHEME

SUMMER 2022

AS (NEW)
FURTHER MATHEMATICS
UNIT 3 FURTHER MECHANICS A
2305U30-1

INTRODUCTION

This marking scheme was used by WJEC for the 2022 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

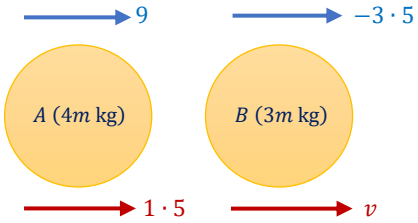
WJEC GCE AS FURTHER MATHEMATICS

UNIT 3 FURTHER MECHANICS A

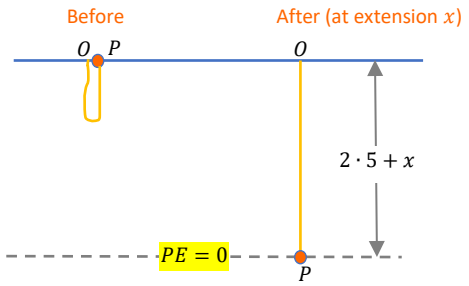
SUMMER 2022 MARK SCHEME

Q1	Solution	Mark	Notes
(a)	Angular velocity $\omega = \frac{v}{r}$ $\omega = \frac{8}{2}$ $\omega = 4 \quad (\text{rad s}^{-1})$	M1 A1 **[2]**	Used cao
(b)	N2L towards centre O Tension in the string $T = 1 \cdot 2a$ $T = 1 \cdot 2 \times \frac{8^2}{2}$ **or** $T = 1 \cdot 2 \times 4^2 \times 2$ $T = 38 \cdot 4 \quad (\text{N})$ or $\frac{192}{5}$	M1 A1 **[2]**	Used with $a = \begin{cases} \frac{v^2}{r} \\ \omega^2 r \end{cases}$ FT their ω from (a)
	Total for Question 1	**4**	

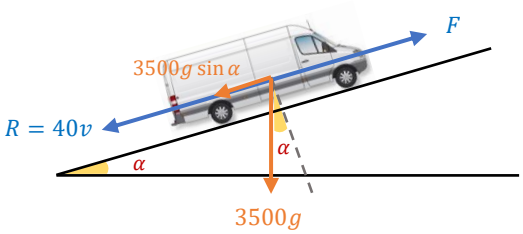
Q2	Solution	Mark	Notes
(a)	Using $\text{KE} = \frac{1}{2}mv^2$ with $m = 60, v = 7 \cdot 8$ $\text{KE} = \frac{1}{2}(60)(7 \cdot 8)^2$ $\text{KE} = 1825 \cdot 2$ (J) or $\frac{9126}{5}$	M1 A1 **[2]**	Used cao
(b)	Using expression for PE **or** KE At start (platform), $\text{PE} = 60g(10)$ $(= 600g = 5880\ \text{J})$ At end (water), $\text{KE} = \frac{1}{2}(60)v^2$ $(= 30v^2)$ Conservation of energy $1825 \cdot 2 + 5880 = 30v^2$ $(7705 \cdot 2 = 30v^2)$ $v^2 = 256 \cdot 84$ or $\frac{6421}{25}$ $v = 16 \cdot 0262 \ldots \approx 16\ (\text{ms}^{-1})$	M1 A1 A1 M1 A1 A1 **[6]**	 Used, all terms, allow sign errors All correct, oe FT KE from (a) Convincing, cso
(c)	Work-energy principle $1825 \cdot 2 + 5880 = \frac{1}{2}(60)(13)^2 + E_{lost}$ $(7705 \cdot 2 = 5070 + E)$ $E_{lost} = 2635 \cdot 2$ (J) or $\frac{13176}{5}$	M1 A1 A1 **[3]**	Used, all terms, allow sign errors All correct, oe FT KE from (a) FT PE from (b) FT their KE and PE
	<u>Alternative Solution</u> Taking a difference in KE $E_{lost} = \frac{1}{2}(60)\left(\frac{6421}{25}\right) - \frac{1}{2}(60)(13)^2$ $E_{lost} = 2635 \cdot 2$ (J) or $\frac{13176}{5}$	 (M1) (A1) (A1) **([3])**	 At least one v^2 correct All correct, oe Accept $\frac{1}{2}(60)(16)^2 = 7680$ $E_{lost} = 2610$ (J) for $v = 16$
	Total for Question 2	**11**	

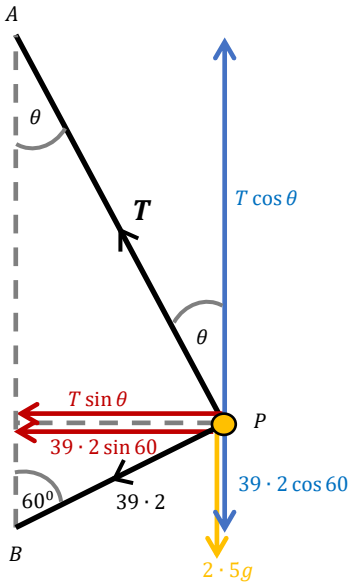
Q3	Solution	Mark	Notes
(a)			
	Conservation of momentum	M1	Attempted. Allow 1 sign error
	$(9)(4m)+(-3\cdot5)(3m)=(1\cdot5)(4m)+(v)(3m)$	A1	All correct
	$25\cdot5=6+3v$		
	$v=6\cdot5\quad(\text{ms}^{-1})$	A1 **[3]**	Convincing
(b)	Restitution	M1	Attempted. Allow 1 sign error
	$6\cdot5-1\cdot5=-e(-3\cdot5-9)$	A1	All correct, oe
	$5=12\cdot5e$		
	$e=\frac{2}{5}$	A1 **[3]**	cao
(c)	Change in momentum $=36$	M1	
	$(4m)(9-1\cdot5)=36\qquad(30m=36)$	A1	Correct equation, oe $(3m)(6\cdot5--3\cdot5)=36$
	$m=1\cdot2$	A1 **[3]**	cao
(d)	Valid reason, eg. Radii are equal Velocities are parallel to line of centres	E1 **[1]**	
	Total for Question 3	**10**	

Q4	Solution	Mark	Notes
(a)	$(9\mathbf{i}+6\mathbf{j}-12\mathbf{k})+(6\mathbf{i}-7\mathbf{j}+3\mathbf{k})+\mathbf{F_3}=0$ $\mathbf{F_3}=-15\mathbf{i}+\mathbf{j}+9\mathbf{k}$ (N)	M1 A1 **[2]**	
(b)	(i) $\mathbf{AB}=\mathbf{r}_B-\mathbf{r}_A=(8\mathbf{i}-5\mathbf{j}-\mathbf{k})-(2\mathbf{i}-9\mathbf{j}+7\mathbf{k})$ $=6\mathbf{i}+4\mathbf{j}-8\mathbf{k}$ $\mathbf{F_1}=\frac{3}{2}\mathbf{AB}$ **or** $\mathbf{AB}=\frac{2}{3}\mathbf{F_1}$ ($\therefore$ parallel)	M1 A1 A1	or $\mathbf{BA}$ oe, cao Convincing
	(ii) Work done by $\mathbf{F_1}=\mathbf{F_1}.\mathbf{AB}$ $=(9\mathbf{i}+6\mathbf{j}-12\mathbf{k}).(6\mathbf{i}+4\mathbf{j}-8\mathbf{k})$ $=(9)(6)+(6)(4)+(-12)(-8)$ $=174$ (J)	M1 A1	Used. FT $\mathbf{AB}$ FT their $\mathbf{AB}$
	(iii) Work done = change in KE $174=\frac{1}{2}(0\cdot5)v^2-0$ $v=26\cdot38(18\ldots)$ (ms^{-1})	M1 A1 **[7]**	FT their '174' $v=\sqrt{696}=2\sqrt{174}$ FT their '174'
	Total for Question 4	**9**	

Q5	Solution	Mark	Notes
(a)			
	Using expression for $\text{PE} = mgh$ **or** $\text{EE} = \frac{\lambda x^2}{2l}$	M1	
	Loss in $\text{PE} = 2g(2\cdot5+x)$ $(= 5g + 2gx)$	A1	
	Gain in $\text{EE} = \frac{\lambda x^2}{2(2\cdot5)} = \frac{30gx^2}{2(2\cdot5)}$ $(= 6gx^2)$	A1	
	Gain in $\text{KE} = \frac{1}{2}(2)v^2$ $(= v^2)$	B1	
	Conservation of energy $v^2 + 6gx^2 = 5g + 2gx$	M1	Used with PE,KE and EE All terms, allow sign errors M0: $\text{PE} = 5g$ alone
	$v^2 = g(5 + 2x - 6x^2)$	A1	Convincing
		[6]	
(b)	At maximum extension, $v = 0$ $0 = g(5 + 2x - 6x^2)$	M1	Used
	$6x^2 - 2x - 5 = 0$ Attempting to solve $x = \frac{2\pm\sqrt{124}}{12}$	m1	$x = \frac{1\pm\sqrt{31}}{6}$ from calculator
	$x = 1\cdot09(4627\ldots)$ (or $x = -0\cdot76(1294\ldots)$)	A1	cao $x = -0\cdot76\ldots$ clearly discarded
		[3]	
(c)	(i) When P attains its maximum speed, $a = 0$ so that Tension in $OP = 2g$	M1	Hooke's Law used with $T = 2g$
	$\frac{30gx}{2\cdot5} = 2g$	A1	
	$x = \frac{1}{6}$ (m)	A1	
	<u>Alternative Solution to (i)</u>		
	(i) Differentiating to find for maximum v^2(or v) $\frac{\mathrm{d}(v^2)}{\mathrm{d}x} = 0$	(M1)	Condone the following incorrect notation $\frac{\mathrm{d}v}{\mathrm{d}x} = g(2 - 12x)$
	$g(2 - 12x) = 0$	(A1)	oe
	$x = \frac{1}{6}$	(A1)	

	(ii) Sub. $x = \frac{1}{6}$ into $v^2 = g(5 + 2x - 6x^2)$ Maximum speed is $7 \cdot 11(57103 \ldots)\ (\text{ms}^{-1})$	M1 A1 **[5]**	FT their $x \geq 0$ $v = \sqrt{\frac{31g}{6}} = \sqrt{\frac{1519}{30}}$. FT their $x \neq 0$ for $v^2 > 0$
	Total for Question 5	**14**	

Q6	Solution	Mark	Notes
(a)			$3500g\left(\frac{3}{49}\right) = 2100$
	$F = \frac{P}{25}$	B1	
	Using N2L up slope $F - R - mg\sin\alpha = ma$	M1 A1	All forces, dim. correct M1: Allow $mg\cos\alpha$ or sign errors, but not both
	$\frac{P}{25} - 40(25) - 3500g\left(\frac{3}{49}\right) = 3500(-0\cdot 2)$	A1	Correct equation FT their F
	$P = 60\,000$ (W) or 60 (kW)	A1 **[5]**	cao
(b)	$F = \frac{40\ \times 1000}{20}$ $(= 2000)$	B1	si
	Using N2L with $a = 0$	M1	All forces, dim. correct M1: Allow $mg\cos\alpha$ or sign errors, but not both
	$F - R - mg\sin\alpha = 0$	A1	
	$2000 - 40(20) - 3500g\sin\alpha = 0$	A1	Correct equation FT their F
	$\sin\alpha = \frac{12}{343} = 0\cdot 03498\ldots$		
	$\alpha = 2^0$	A1 **[5]**	cao
	Total for Question 6	**10**	

Q7	Solution	Mark	Notes
(a)			$\sin\theta = 0 \cdot 6$ $\cos\theta = 0 \cdot 8$
	Resolving vertically,	M1	All forces, dim. correct
	$T\cos\theta = (39 \cdot 2)\cos 60 + 2 \cdot 5g$	A1	-1 each error
	$T(0 \cdot 8) = (39 \cdot 2)(0 \cdot 5) + (2 \cdot 5)(9 \cdot 8)$	A1	
	$T = 55 \cdot 125$ (N)	A1	cao
		[4]	
(b)	Using N2L towards C,	M1	All forces, dim. correct
	$T\sin\theta + (39 \cdot 2)\sin 60 = 2 \cdot 5a$	A1	Correct equation
	$(55 \cdot 125)(0 \cdot 6) + (39 \cdot 2)\left(\frac{\sqrt{3}}{2}\right) = (2 \cdot 5)\,\omega^2 r$	m1	$a = \begin{cases} \frac{v^2}{r} \\ \omega^2 r \end{cases}$
	$(55 \cdot 125)(0 \cdot 6) + (39 \cdot 2)\left(\frac{\sqrt{3}}{2}\right) = (2 \cdot 5)\,\omega^2 (0 \cdot 9)$	B1	$r = 1 \cdot 5\sin\theta$ $r = 1 \cdot 5 \times 0 \cdot 6 = 0 \cdot 9$
	$\omega^2 = 29 \cdot 78808\ldots$		
	$\omega = 5 \cdot 45\ (78463\ldots)$ $(\text{rad}\ s^{-1})$	A1	cao
		[5]	
(c)	$v = \omega r$		
	$v = 5 \cdot 45\ldots \times 0 \cdot 9$ $v = 4 \cdot 91206\ldots$	M1	FT ω and $r \neq 1 \cdot 5$
	$KE = \frac{1}{2}(2 \cdot 5)(4 \cdot 91206\ldots)^2$	m1	FT v
	$KE = 30 \cdot 16\ (0438\ldots)$ (J)	A1	cao
		[3]	
	Total for Question 7	**12**	