# GCE AS/A LEVEL

2305U10-1

Z22-2305U10-1

**MONDAY, 16 MAY 2022 – AFTERNOON**

# FURTHER MATHEMATICS – AS unit 1
## FURTHER PURE MATHEMATICS A

1 hour 30 minutes

**ADDITIONAL MATERIALS**

In addition to this examination paper, you will need:
- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

**INSTRUCTIONS TO CANDIDATES**

Use black ink or black ball-point pen.
Answer **all** questions.
Sufficient working must be shown to demonstrate the **mathematical** method employed.
Answers without working may not gain full credit.
Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

**INFORMATION FOR CANDIDATES**

The maximum mark for this paper is 70.
The number of marks is given in brackets at the end of each question or part-question.
You are reminded of the necessity for good English and orderly presentation in your answers.

**Reminder**: Sufficient working must be shown to demonstrate the **mathematical** method employed.

**1.** The complex numbers $z$, $w$ are given by $z = 3 - 4\text{i}$, $w = 2 - \text{i}$.

(a) (i) Find the modulus and argument of $zw$.

(ii) Express $zw$ in the form $r(\cos\theta + \text{i}\sin\theta)$. [5]

(b) The complex numbers $v$, $w$, $z$ satisfy the equation $\dfrac{1}{v} = \dfrac{1}{w} - \dfrac{1}{z}$.

Find $v$ in the form $a + \text{i}b$, where $a$, $b$ are real. [5]

(c) The complex conjugate of $v$ is denoted by $\bar{v}$.

Show that $v\bar{v} = k$, where $k$ is a real number whose value is to be determined. [2]

**2.** (a) The matrices $\mathbf{A}$ and $\mathbf{B}$ are defined by

$$\mathbf{A} = \begin{pmatrix} 3 & 4 \\ -1 & -2 \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} -11 \\ 7 \end{pmatrix}.$$

Given that $\mathbf{AX} = \mathbf{B}$, find the matrix $\mathbf{X}$. [4]

(b) (i) Find the 2 × 2 matrix, $\mathbf{T}$, which represents a reflection in the line $y = -2x$.

(ii) The images of the points $C(2, 7)$ and $D(3, 1)$, under $\mathbf{T}$, are $E$ and $F$ respectively.

Find the coordinates of the midpoint of $EF$. [7]

**3.** The vector equation of the line $L$ is given by

$$\mathbf{r} = -\mathbf{i} + 2\mathbf{j} - 6\mathbf{k} + \lambda(4\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}).$$

The Cartesian equation of the plane $\Pi$ is given by

$$3x + 8y - 9z = 0.$$

Find the Cartesian coordinates of the point of intersection of $L$ and $\Pi$. [5]

**4.** The positive integer $N$ is such that $1^2 + 2^2 + 3^2 + \dots + N^2 = (3N - 2)^2$.

Write down and simplify an equation satisfied by $N$. Hence find the possible values of $N$. [7]

**5.** (a) The complex number $z$ is represented by the point $P(x, y)$ in an Argand diagram. Given that

$$|z - 3 + 2\mathrm{i}| = |z - 3|,$$

find the equation of the locus of $P$. [3]

(b) Give a geometric interpretation of the locus of $P$. [1]

**6.** The roots of the cubic equation

$$2x^3 + px^2 - 126x + q = 0$$

form a geometric progression with common ratio −3.

Find the possible values of $p$ and $q$, giving your answers in surd form. [8]

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**7.** The vector equations of the lines $L_1$, $L_2$, $L_3$ are given by

$$\mathbf{r} = 3\mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(2\mathbf{i} + n\mathbf{j} + \mathbf{k}),$$

$$\mathbf{r} = 5\mathbf{i} - 3\mathbf{j} - 4\mathbf{k} + \mu(3\mathbf{i} + \mathbf{j} - 3\mathbf{k}),$$

$$\mathbf{r} = 6\mathbf{i} - 3\mathbf{j} + 2\mathbf{k} + v(p\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}),$$

respectively, where $n$ and $p$ are constants.

The line $L_1$ is perpendicular to the line $L_2$. The line $L_1$ is also perpendicular to the line $L_3$.

(a) Show that the value of $n$ is –3 and find the value of $p$. [3]

(b) Find the acute angle between the lines $L_2$ and $L_3$. [4]

**8.** The point $(x, y, z)$ is rotated through 60° anticlockwise around the $z$-axis.
After rotation, the value of the $x$-coordinate is equal to the value of the $y$-coordinate.

Show that $y = \left(a + \sqrt{b}\right)x$, where $a$, $b$ are integers whose values are to be determined. [7]

**9.** (a) Given that $A_r = \dfrac{1}{r+1} - \dfrac{2}{r+2} + \dfrac{1}{r+3}$, show that $A_r$ can be expressed as $\dfrac{2}{(r+1)(r+2)(r+3)}$. [2]

(b) Hence, show that $\displaystyle\sum_{r=1}^{n} \frac{2}{(r+1)(r+2)(r+3)} = \frac{1}{6} - \frac{1}{(n+2)(n+3)}$. [5]

(c) Find the ratio of $\displaystyle\sum_{r=1}^{5} A_r : \sum_{r=1}^{10} A_r$, giving your answer in its simplest form. [2]

**END OF PAPER**