GCE AS/A LEVEL

2305U30-1

Z22-2305U30-1

THURSDAY, 26 MAY 2022 – AFTERNOON

FURTHER MATHEMATICS – AS unit 3
FURTHER MECHANICS A

1 hour 30 minutes

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ADDITIONAL MATERIALS

In addition to this examination paper, you will need:
- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.
Answer **all** questions.
Take g as $9{\cdot}8\text{ms}^{-2}$.
Sufficient working must be shown to demonstrate the **mathematical** method employed.
Answers without working may not gain full credit.
Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

INFORMATION FOR CANDIDATES

The maximum mark for this paper is 70.
The number of marks is given in brackets at the end of each question or part-question.
You are reminded of the necessity for good English and orderly presentation in your answers.

Reminder: Sufficient working must be shown to demonstrate the **mathematical** method employed.

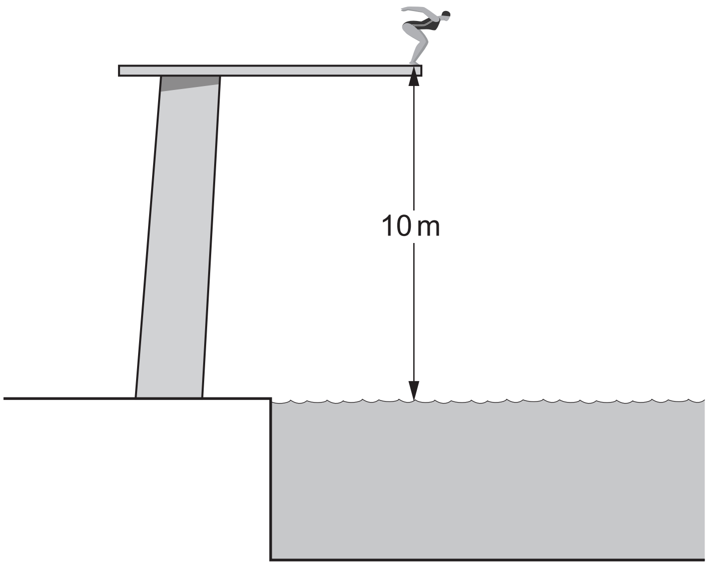
1. A particle of mass 1·2 kg is attached to one end of a light inextensible string of length 2 m. The other end of the string is fixed to a point *O* on a smooth horizontal surface. With the string taut, the particle moves on the surface with constant speed 8 ms^{-1} in a horizontal circle with centre *O*.

(a) Find the angular velocity of the particle about *O*. [2]

(b) Calculate the tension in the string. [2]

2. The diagram below shows a woman standing at the end of a diving platform. She is about to dive into the water below.

The woman has mass 60 kg and she may be modelled as a particle positioned at the end of the platform which is 10 m above the water.



When the woman dives, she projects herself from the platform with a speed of 7·8 ms^{-1}.

(a) Find the kinetic energy of the woman when she leaves the platform. [2]

(b) Initially, the situation is modelled ignoring air resistance. By using conservation of energy, show that the model predicts that the woman enters the water with an approximate speed of 16 ms^{-1}. [6]

(c) Suppose that this model is refined to include air resistance so that the speed with which the woman enters the water is now predicted to be 13 ms^{-1}.
Determine the amount of energy lost to air resistance according to the refined model. [3]

3. Two spheres *A* and *B*, of equal radii, are moving towards each other on a smooth horizontal surface and collide directly. Sphere *A* has mass $4m$ kg and sphere *B* has mass $3m$ kg. Just before the collision, *A* has speed $9\,\text{ms}^{-1}$ and *B* has speed $3{\cdot}5\,\text{ms}^{-1}$. Immediately after the collision, *A* has speed $1{\cdot}5\,\text{ms}^{-1}$ in the direction of its original motion.

(a) Show that the speed of *B* immediately after the collision is $6{\cdot}5\,\text{ms}^{-1}$. [3]

(b) Calculate the coefficient of restitution between *A* and *B*. [3]

(c) Given that the magnitude of the impulse exerted by *B* on *A* is 36 Ns, find the value of m. [3]

(d) Give a reason why it is not necessary to model the spheres as particles in this question. [1]

4. A particle *P* of mass 0·5 kg is in equilibrium under the action of three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$.

$$\mathbf{F}_1 = (9\mathbf{i} + 6\mathbf{j} - 12\mathbf{k})\,\text{N} \quad \text{and} \quad \mathbf{F}_2 = (6\mathbf{i} - 7\mathbf{j} + 3\mathbf{k})\,\text{N}.$$

(a) Find the force $\mathbf{F}_3$. [2]

(b) Forces $\mathbf{F}_2$ and $\mathbf{F}_3$ are removed so that *P* moves in a straight line *AB* under the action of the single force $\mathbf{F}_1$. The points *A* and *B* have position vectors $(2\mathbf{i} - 9\mathbf{j} + 7\mathbf{k})$ m and $(8\mathbf{i} - 5\mathbf{j} - \mathbf{k})$ m respectively. The particle *P* is initially at rest at *A*.

(i) Verify that $\mathbf{F}_1$ acts parallel to the vector **AB**.

(ii) Find the work done by the force $\mathbf{F}_1$ as the particle moves from *A* to *B*.

(iii) By using the work-energy principle, find the speed of *P* as it reaches *B*. [7]

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TURN OVER

5. One end of a light elastic string, of natural length 2·5 m and modulus of elasticity $30g$ N, is fixed to a point *O*. A particle *P*, of mass 2 kg, is attached to the other end of the string. Initially, *P* is held at rest at the point *O*. It is then released and allowed to fall under gravity.

(a) Show that, while the string is taut,

$$v^2 = g\left(5 + 2x - 6x^2\right),$$

where v ms^{-1} denotes the velocity of the particle when the extension in the string is x m. [6]

(b) Calculate the maximum extension of the string. [3]

(c) (i) Find the extension of the string when *P* attains its maximum speed.

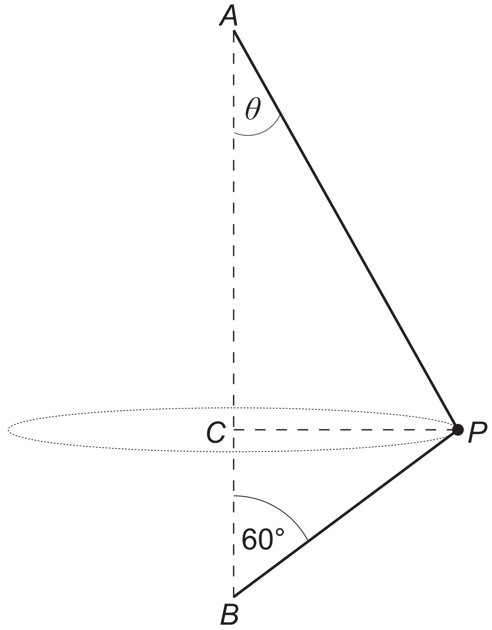
(ii) Hence determine the maximum speed of *P*. [5]

6. A vehicle of mass 3500 kg is moving up a slope inclined at an angle α to the horizontal. When the vehicle is travelling at a velocity of v ms^{-1}, the resistance to motion can be modelled by a variable force of magnitude $40v$ N.

(a) Given that $\sin\alpha = \frac{3}{49}$, calculate the power developed by the engine at the instant when the speed of the vehicle is 25 ms^{-1} and its deceleration is 0·2 ms^{-2}. [5]

(b) When the vehicle's engine is working at a constant rate of 40 kW, the maximum speed that can be maintained up the slope is 20 ms^{-1}. Find the value of α. Give your answer in degrees, correct to one significant figure. [5]

7. The diagram below shows a particle *P*, of mass 2·5 kg, attached by means of two light inextensible strings fixed at points *A* and *B*. Point *A* is vertically above point *B*. *BP* makes an angle of 60° with the upward vertical and *AP* is inclined at an angle θ to the downward vertical where $\cos\theta = 0{\cdot}8$. The particle *P* describes a horizontal circle with constant angular speed ω radians per second about centre *C* with both strings taut.



The tension in the string *BP* is 39·2 N.

(a) Calculate the tension in the string *AP*. [4]

(b) Given that the length of the string *AP* is 1·5 m, find the value of ω. [5]

(c) Calculate the kinetic energy of *P*. [3]

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